qIND-qCPA (In)security of CBC, CFB, OFB and CTR

PQCrypto 2023

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Reminders on modes of operation

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Figure: Original image

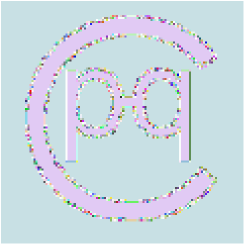

Figure: Encrypted image

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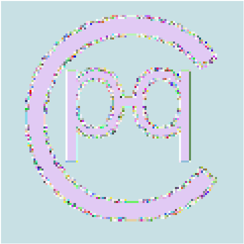
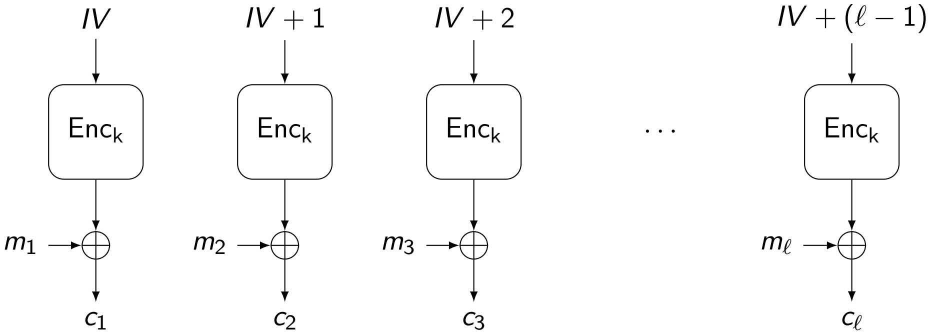


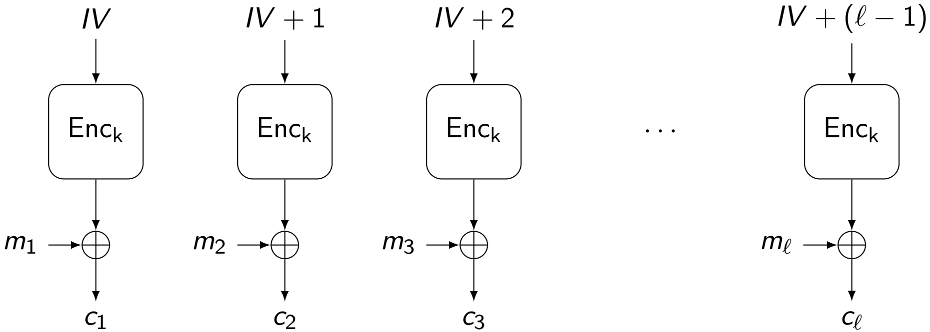
Figure: Encrypted image

Modes of operation allow us to securely encrypt long messages.

Example: CTR mode



Example: CTR mode



In particular, it acts as a stream cipher: $\mathsf{Enc}_\mathsf{k}^{\mathsf{CTR}}(m) = m \oplus s$ for a (pseudo)random s.

History of the security of these modes

	CTR/OFB	CBC/OFB with PRP	CBC/OFB with qPRP
IND-CPA			

IND-CPA Classical learning queries, Classical challenge queries

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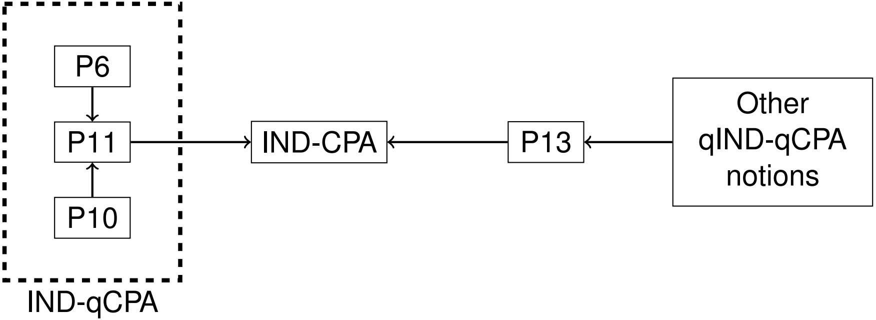

Previous work

Our work

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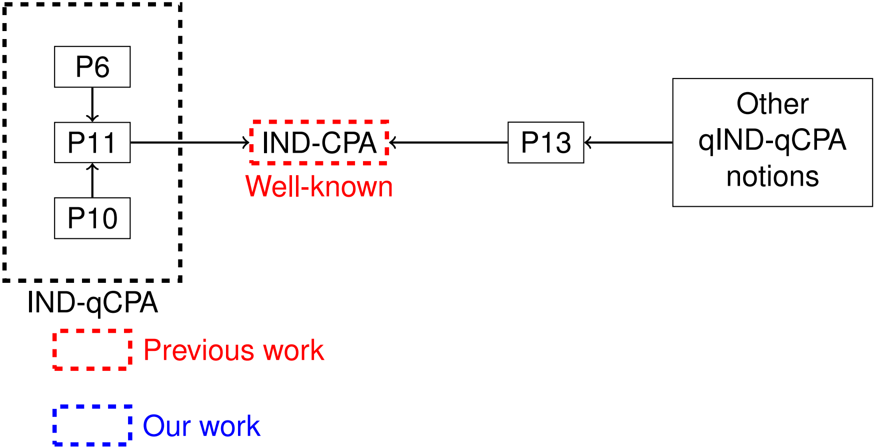

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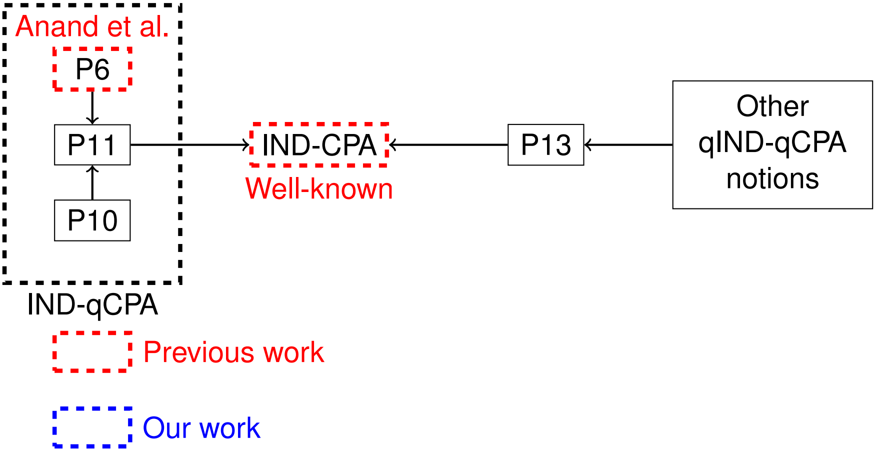

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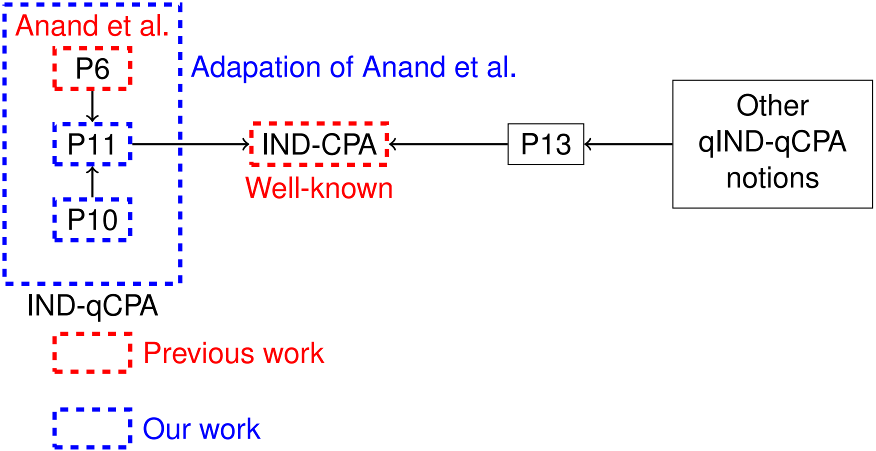

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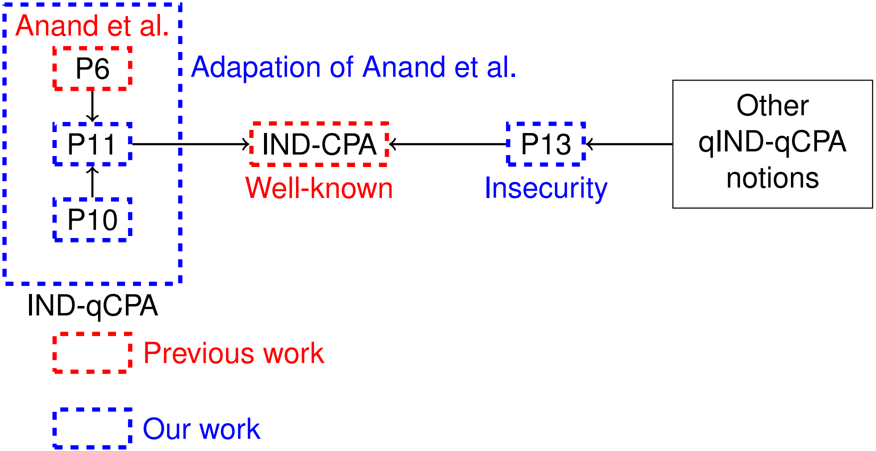

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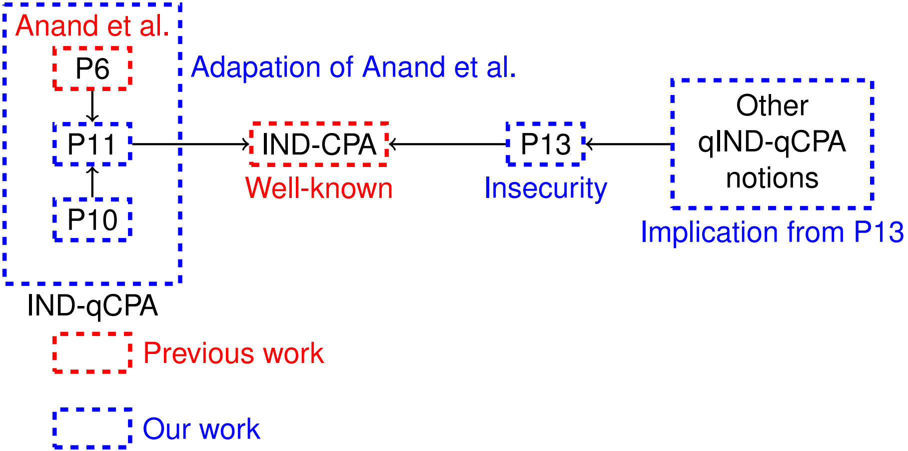
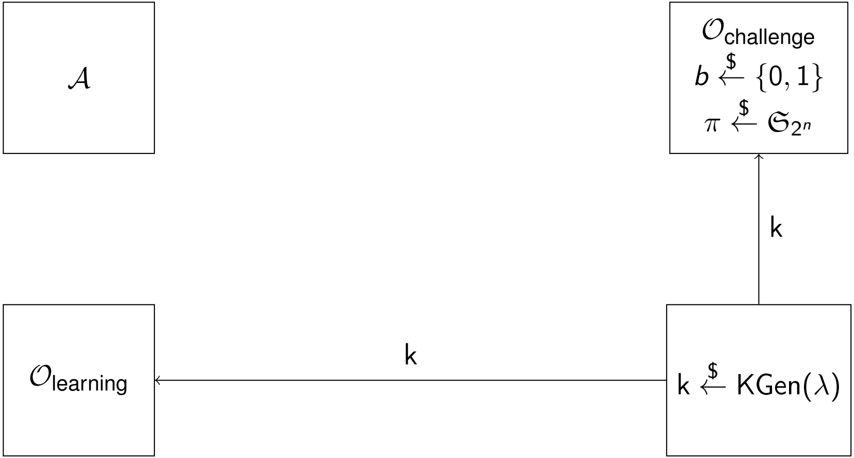


qIND-qCPA-P13 security notion



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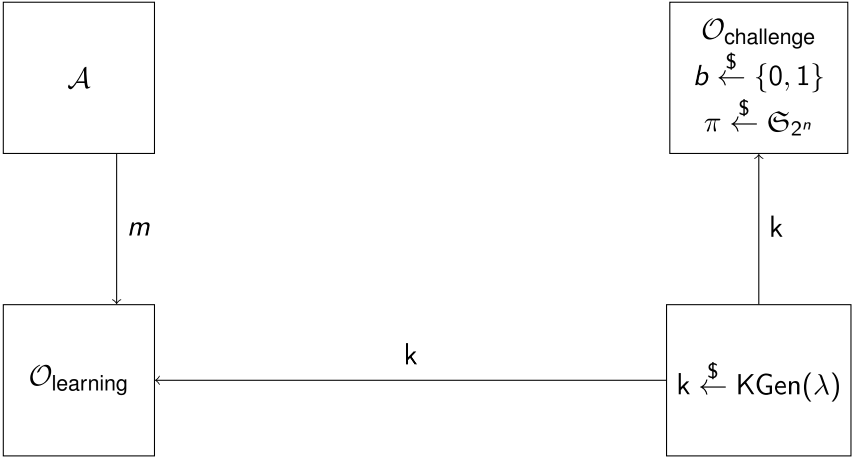

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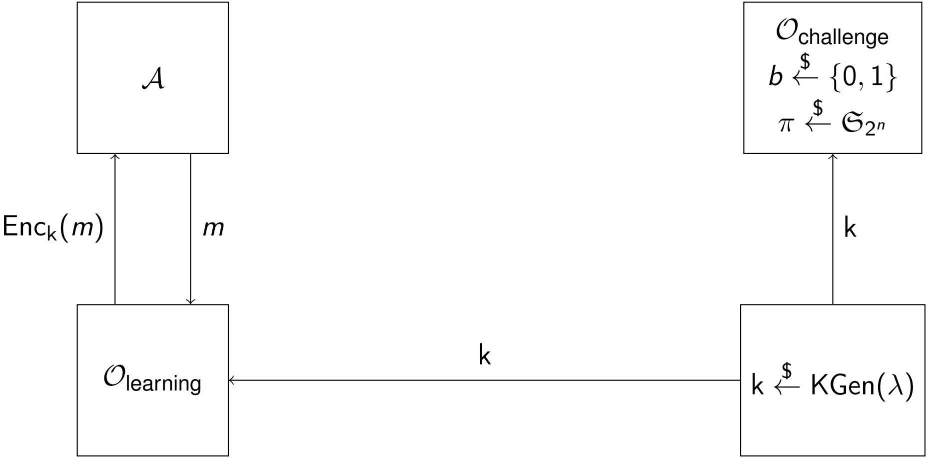

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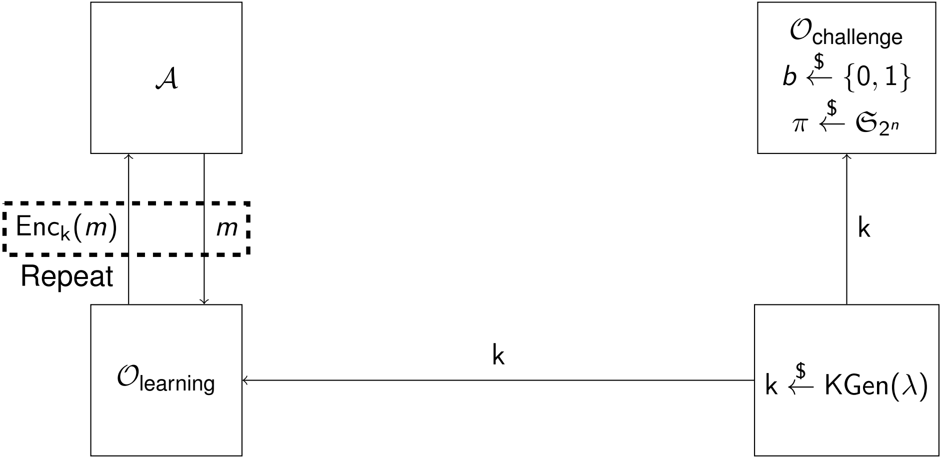

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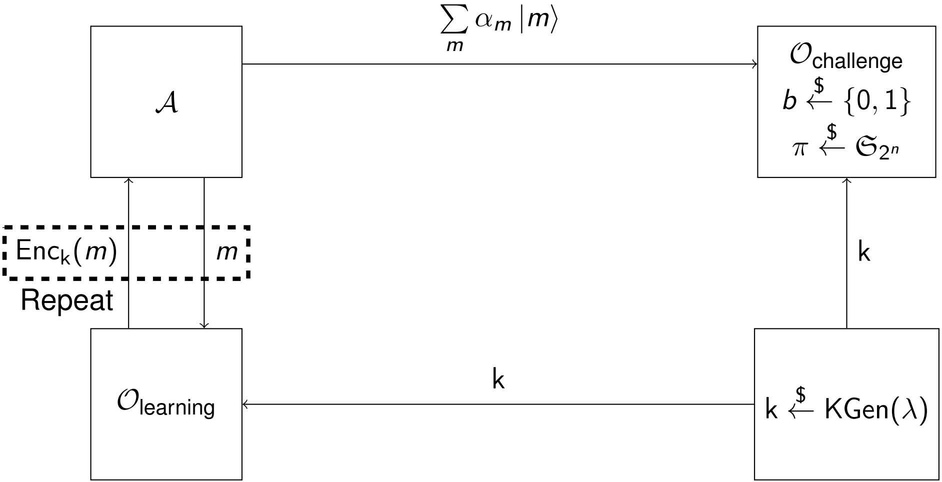

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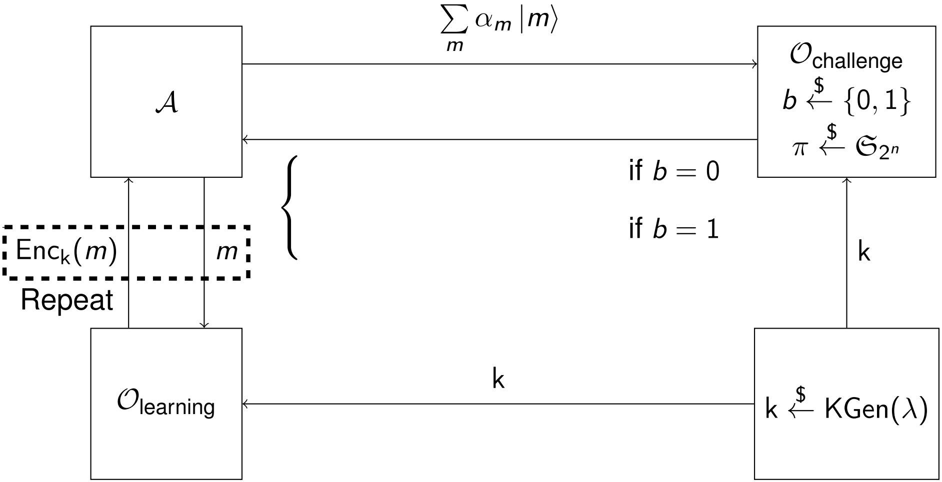

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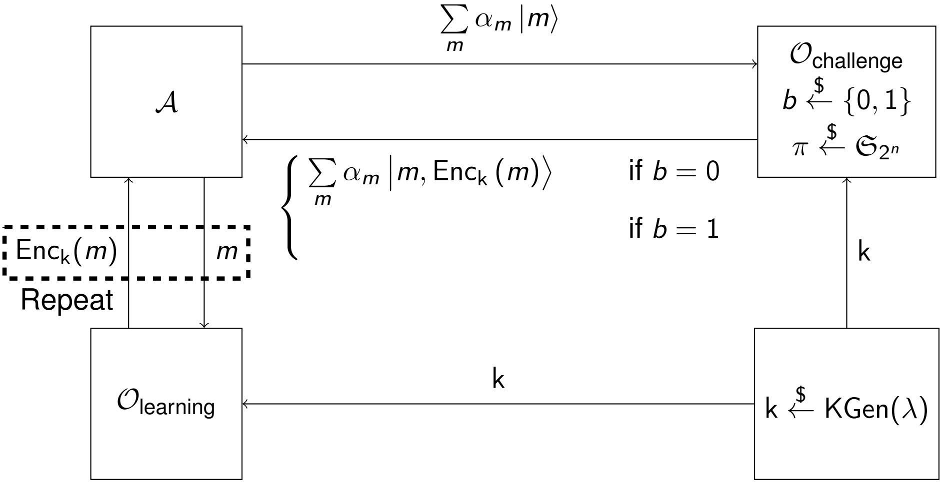

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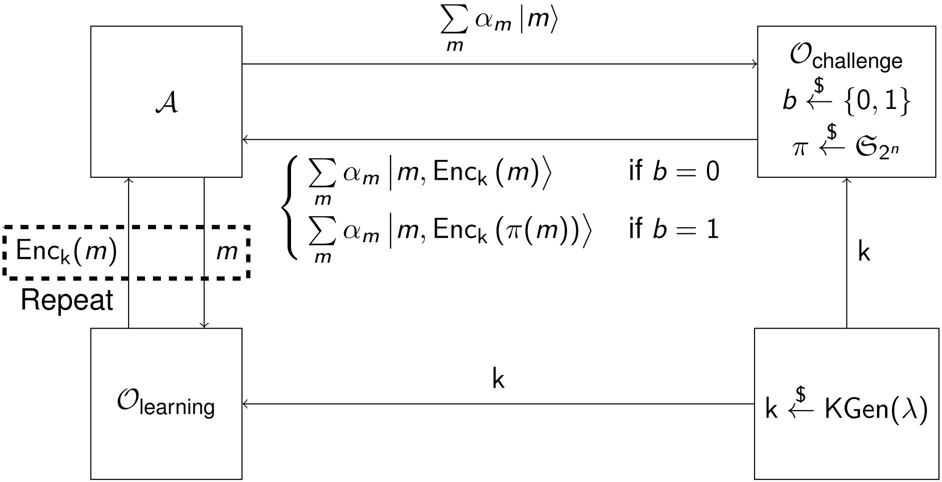

Example: qIND-qCPA-P13 insecurity of a stream cipher

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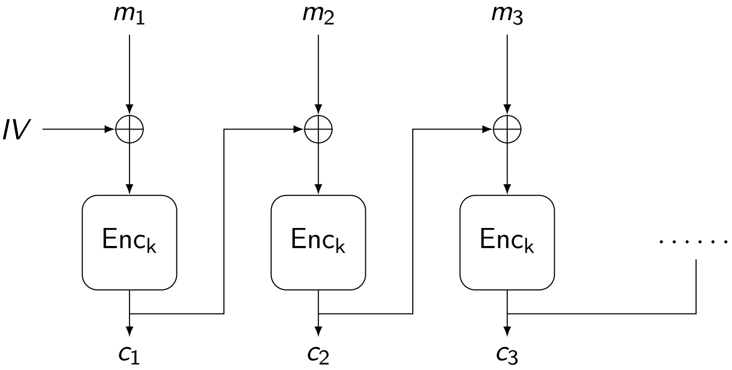

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Invalidates most modes of operation. . .

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Relevance of the qIND-qCPA security notions

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Relevance of the qIND-qCPA security notions

- Attacks are generic: how bad is it for a scheme to be qIND-qCPA-P13-insecure?
- No equivalent semantic security notion for most qIND-qCPA notions

Takeaway: we need to perform more research to define a useful qIND-qCPA notion.

Thank you!

What are the qIND-qCPA security notions?

Two oracles for the IND-CPA game:

$$\begin{aligned} \textbf{Query} &\quad m \rightarrow \mathsf{Enc}_{\mathsf{k}}(m) \\ \textbf{Challenge} &\quad m_0, m_1 \rightarrow \mathsf{Enc}_{\mathsf{k}}\left(m_b\right) \end{aligned}$$

What are the qIND-qCPA security notions?

Two oracles for the IND-CPA game:

Query $m \to \mathrm{Enc}_k(m)$

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Challenge type To choose from:

- Left-or-Right

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Two oracles for the IND-CPA game:

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Number of challenge queries A single or $\mathsf{poly}(\lambda)$

Oracle type To choose from:

- Classical

What are the qIND-qCPA security notions?

Two oracles for the IND-CPA game:

Query $|x, y\rangle \to |x, y \oplus \mathsf{Enc_k}(x)\rangle$

Challenge $|x, y\rangle \to |x, y \oplus \mathsf{Enc_k}\left(\pi^b(x)\right)\rangle$

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Number of challenge queries A single or $\mathsf{poly}(\lambda)$

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- Embedding

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$\Longrightarrow$ Many notions, some of them being equivalent.

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$\implies$ Many notions, some of them being equivalent.

$\implies$ 14 different qIND-qCPA notions

Bibliography I

Anand, Mayuresh Vivekanand et al. “Post-Quantum Security of the CBC, CFB, OFB, CTR, and XTS Modes of Operation”. In: *Post-Quantum Cryptography - 7th International Workshop, PQCrypto 2016*. Ed. by Tsuyoshi Takagi. Fukuoka, Japan: Springer, Heidelberg, Germany, Feb. 2016, pp. 44–63. DOI: 10.1007/978-3-319-29360-8_4.

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