

Post-Quantum Signatures from Multiparty Computation: Recent Advances

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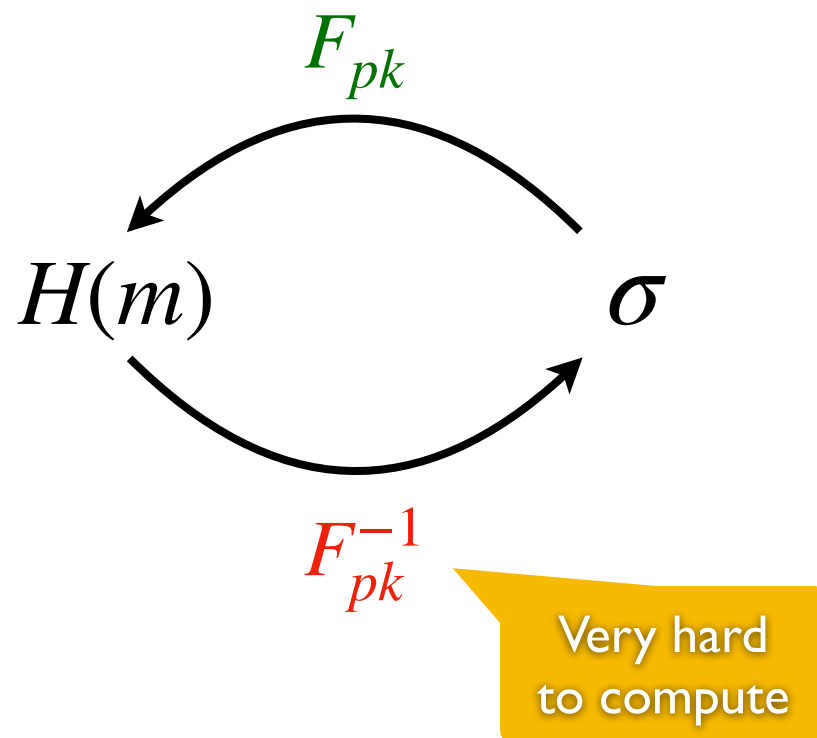
- Introduction
- MPC-in-the-Head: general principle
- From MPC-in-the-Head to signatures
- Optimisations and variants
- Related works
- Conclusion

Some figures used in the following slides have been realised by Matthieu Rivain (CryptoExperts).

Introduction

How to build signature schemes?

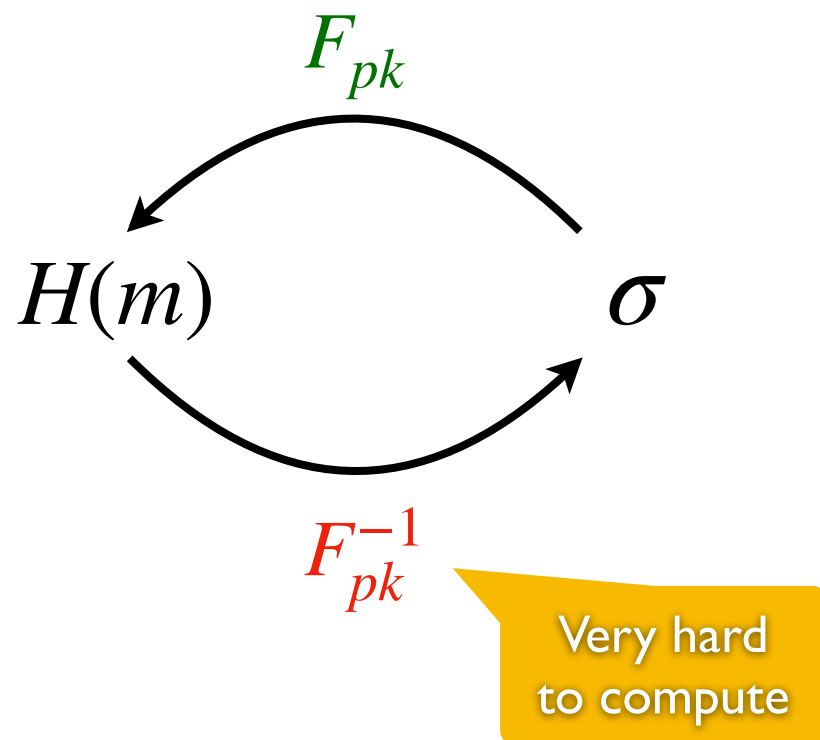
Hash & Sign



- Short signatures
- “Trapdoor” in the public key

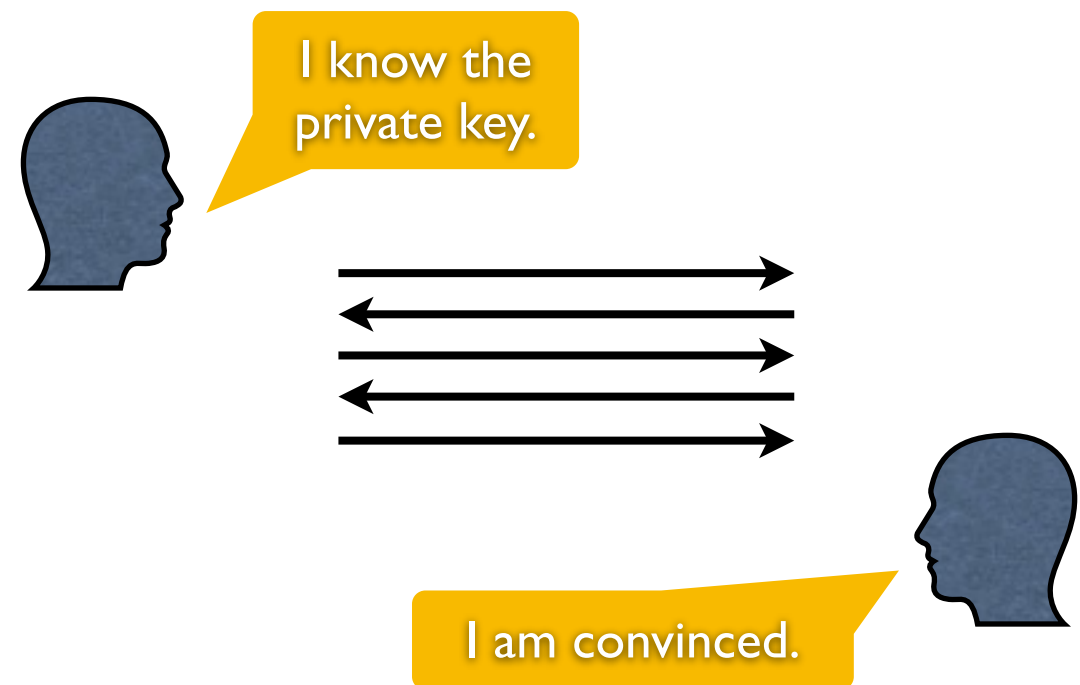
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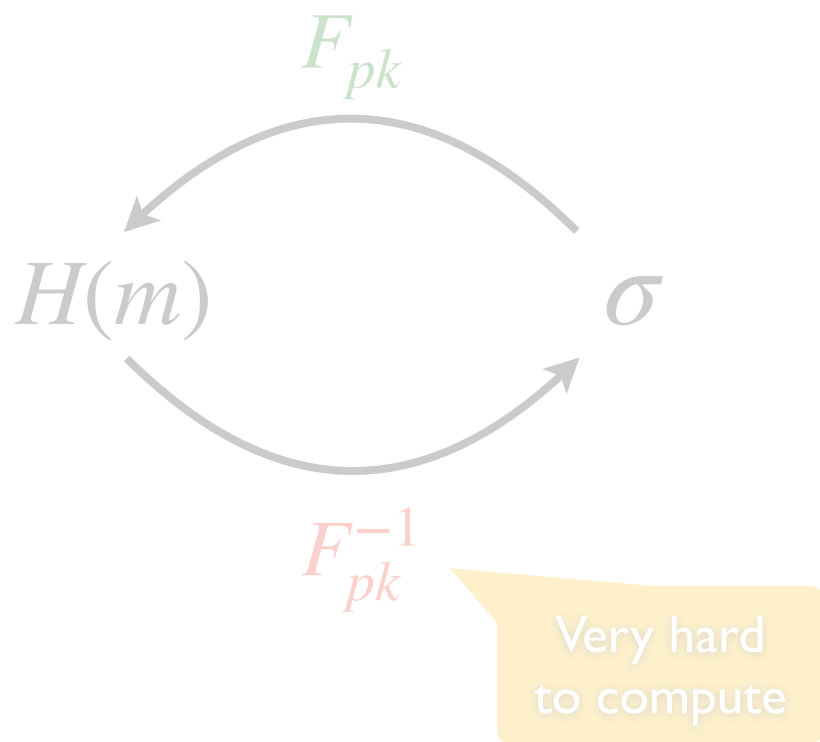
From a zero-knowledge proof



- Large(r) signatures
- Short public key

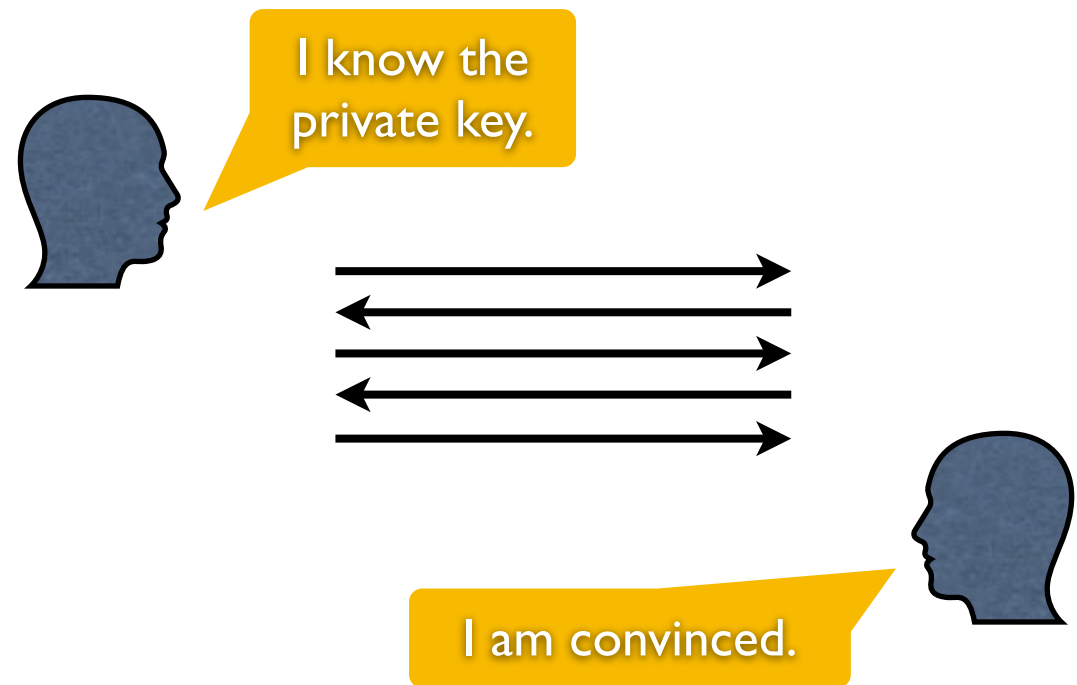
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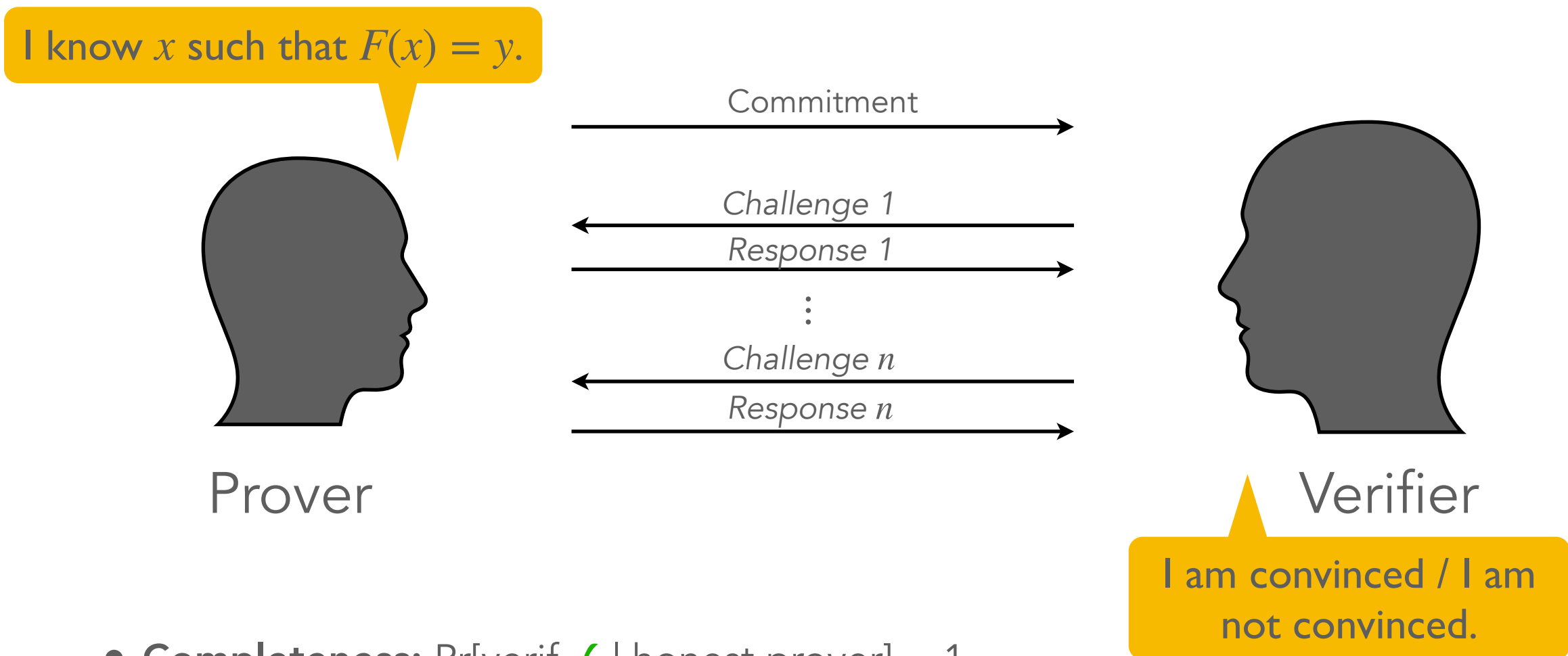
- Short signatures
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From a zero-knowledge proof



- Large(r) signatures
- Short public key

Proof of knowledge



- **Completeness:** $\Pr[\text{verif } \checkmark \mid \text{honest prover}] = 1$
- **Soundness:** $\Pr[\text{verif } \checkmark \mid \text{malicious prover}] \leq \epsilon$ (e.g. 2^{-128})
- **Zero-knowledge:** verifier learns nothing on x

MPC in the Head

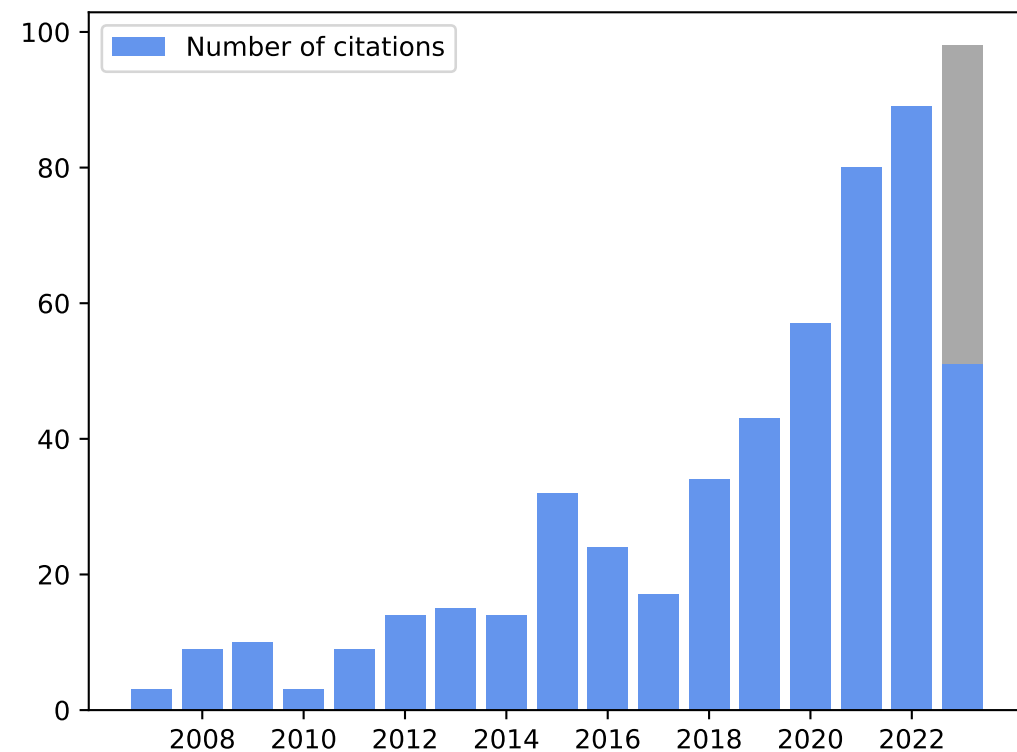
- **[IKOS07]** Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, Amit Sahai: "Zero-knowledge from secure multiparty computation" (STOC 2007)
- Turn an MPC protocol into a zero knowledge proof of knowledge
- **Generic:** can be apply to any cryptographic problem

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Figure: Number of citations to [IKOS07] by year

Source: Google Scholar

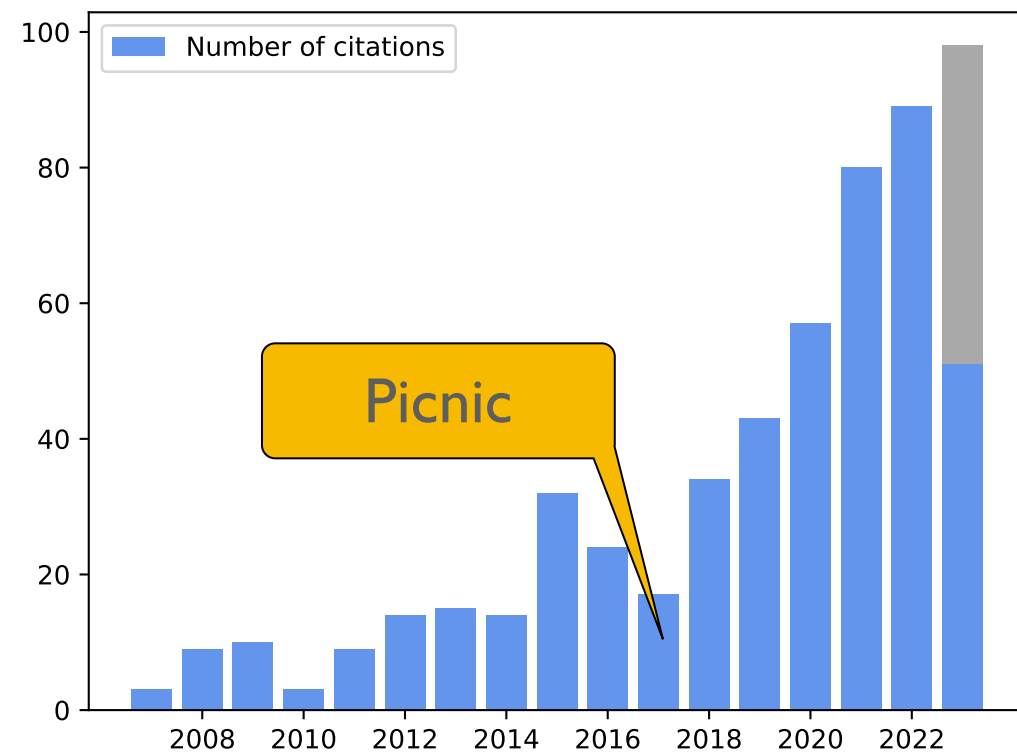


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- [IKOS07] Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, Amit Sahai: "Zero-knowledge from secure multiparty computation" (STOC 2007)
- Turn an MPC protocol into a zero knowledge proof of knowledge
- **Generic:** can be apply to any cryptographic problem
- Convenient to build (candidate) **post-quantum signature** schemes
- **Picnic:** submission to NIST (2017)
- First round of recent NIST call: 8 MPCitH schemes / 40 submissions

AIMer

MQOM

Biscuit

PERK

MIRA

RYDE

MiRitH

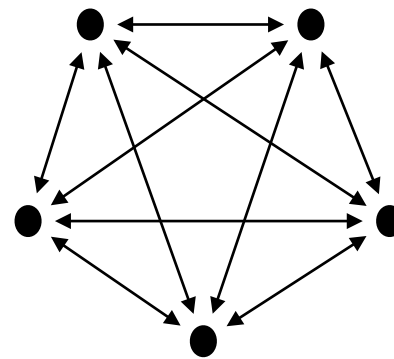
SDitH

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

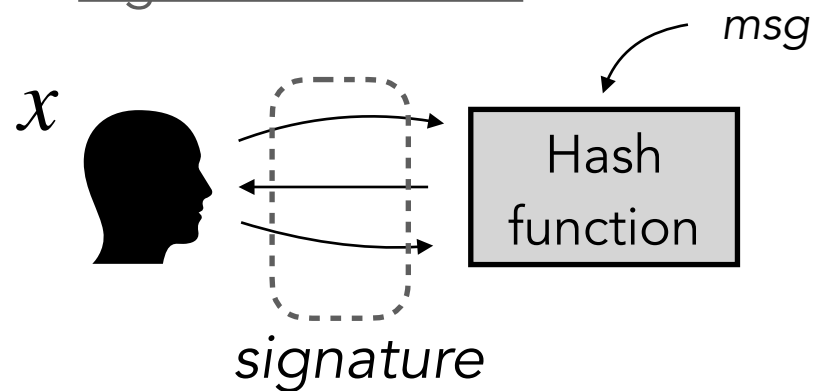
Multiparty computation (MPC)



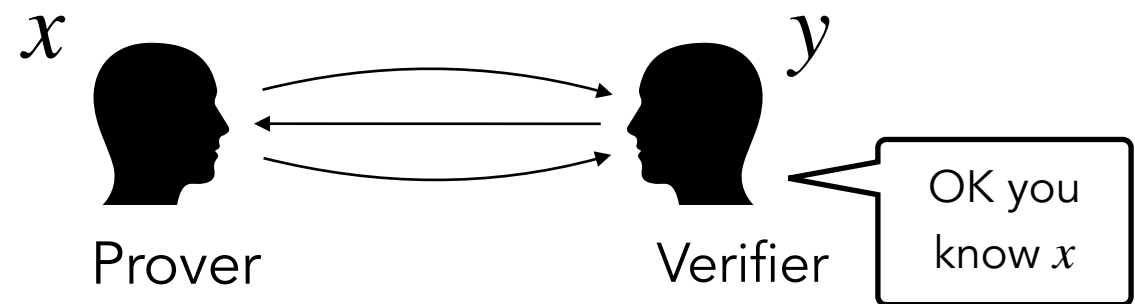
Input sharing $[[x]]$
Joint evaluation of:

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

Signature scheme



Zero-knowledge proof

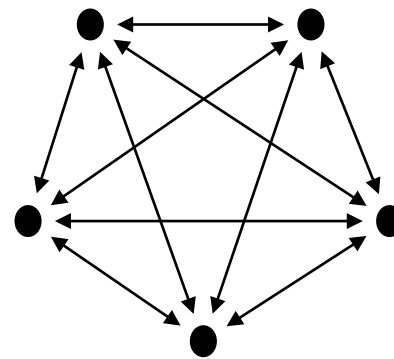


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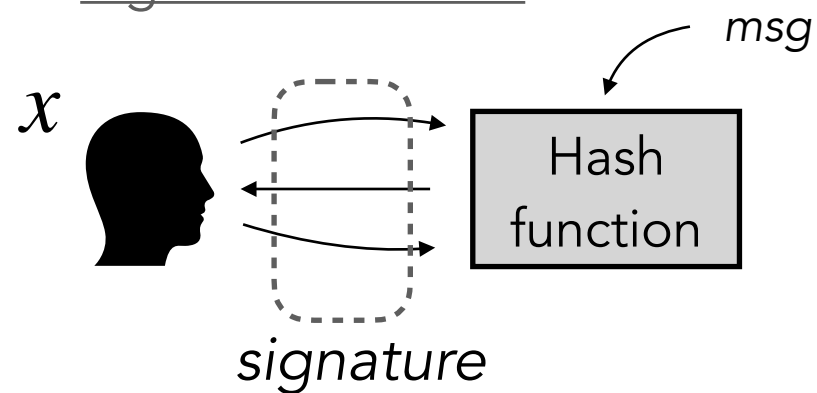
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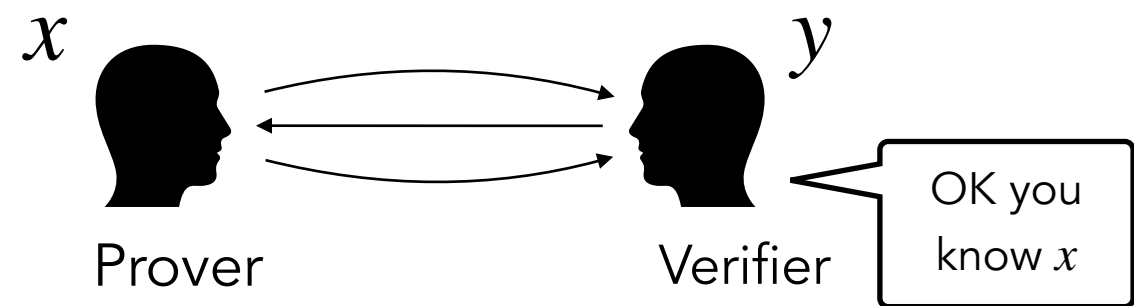
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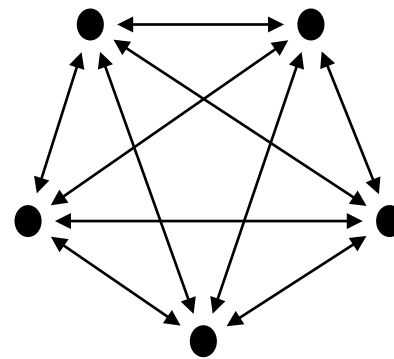
$$[[x]] = ([[x]]_1, \dots, [[x]]_N) \quad \text{s.t.} \quad x = [[x]]_1 + \dots + [[x]]_N$$

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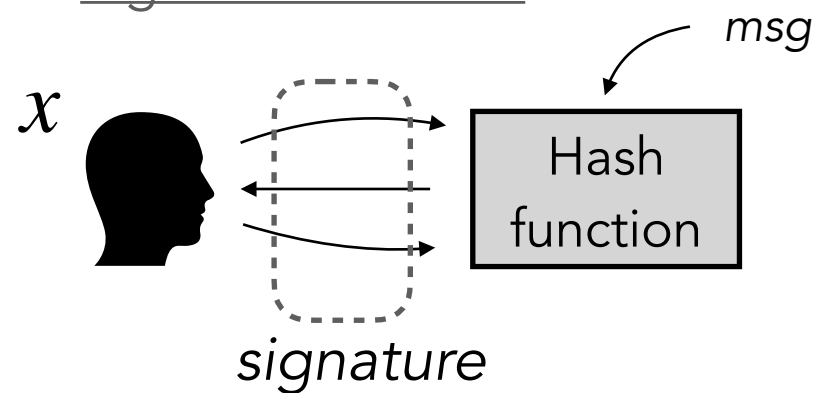
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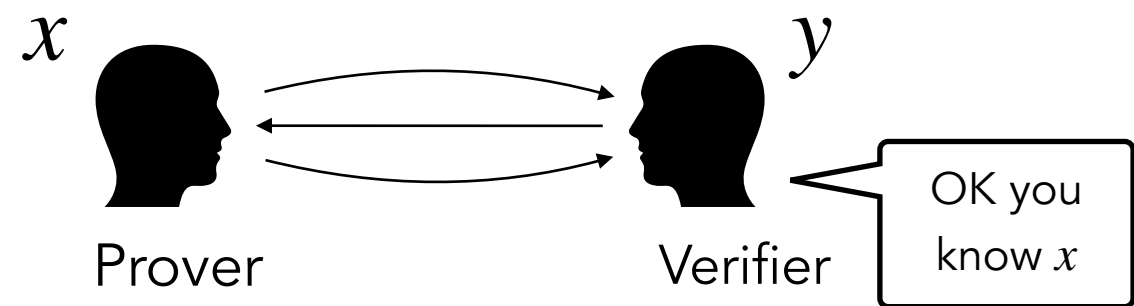
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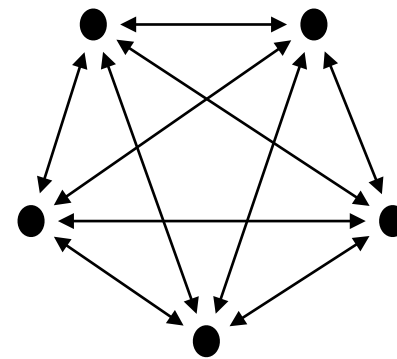


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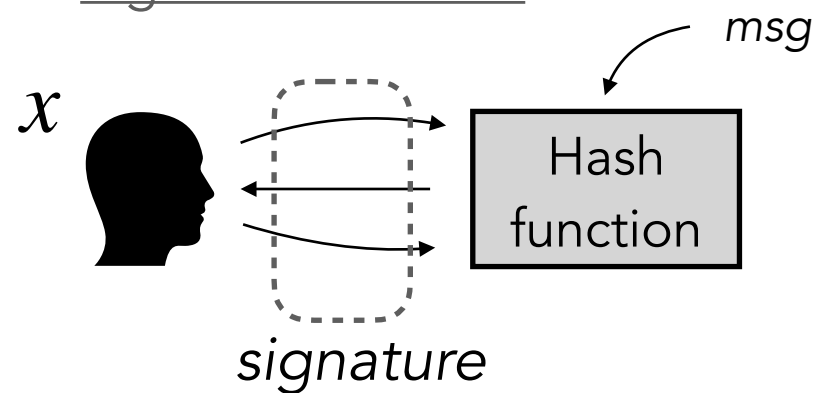
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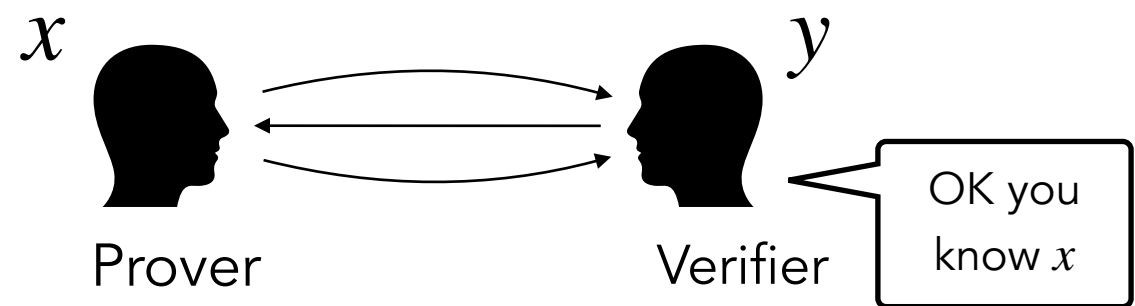
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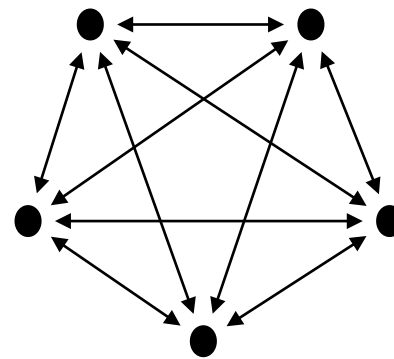


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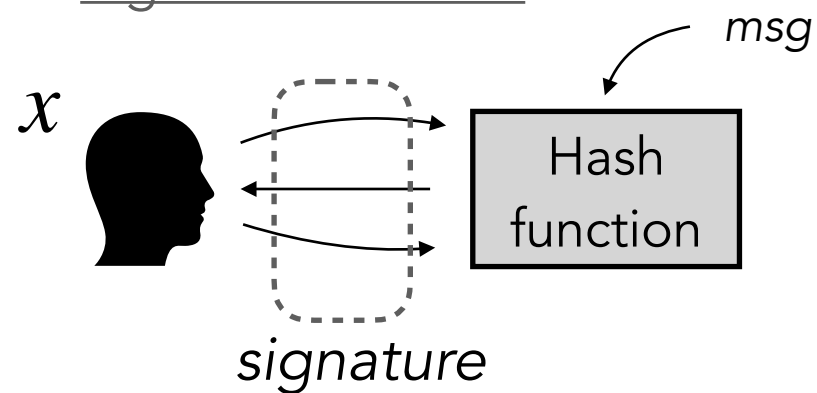
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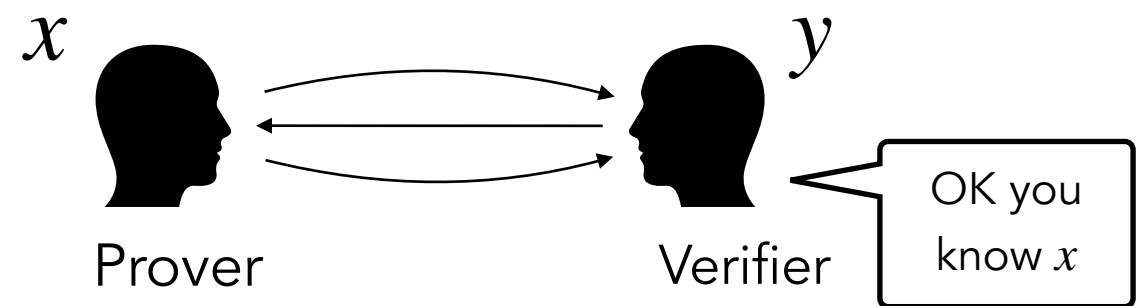
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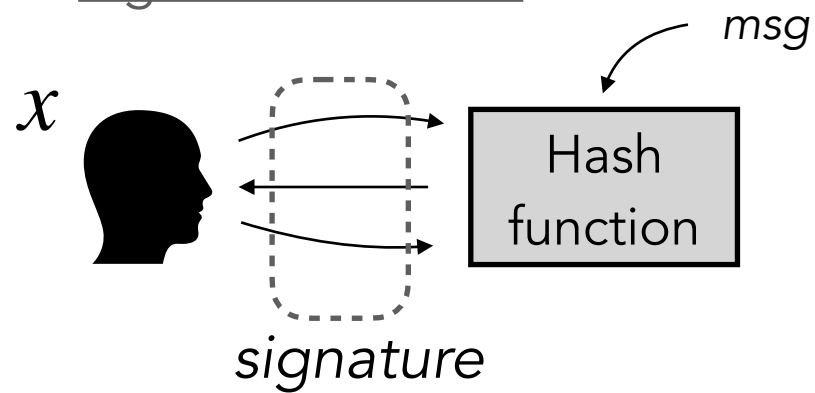


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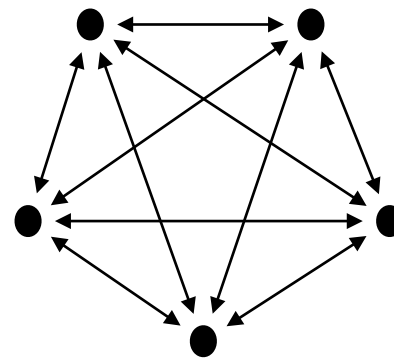
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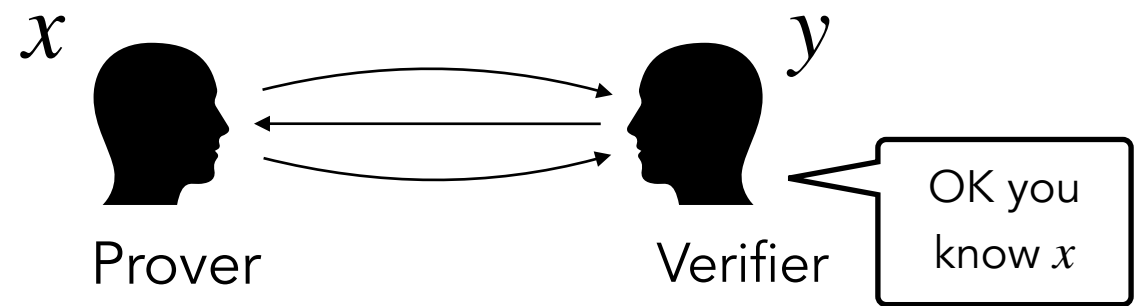


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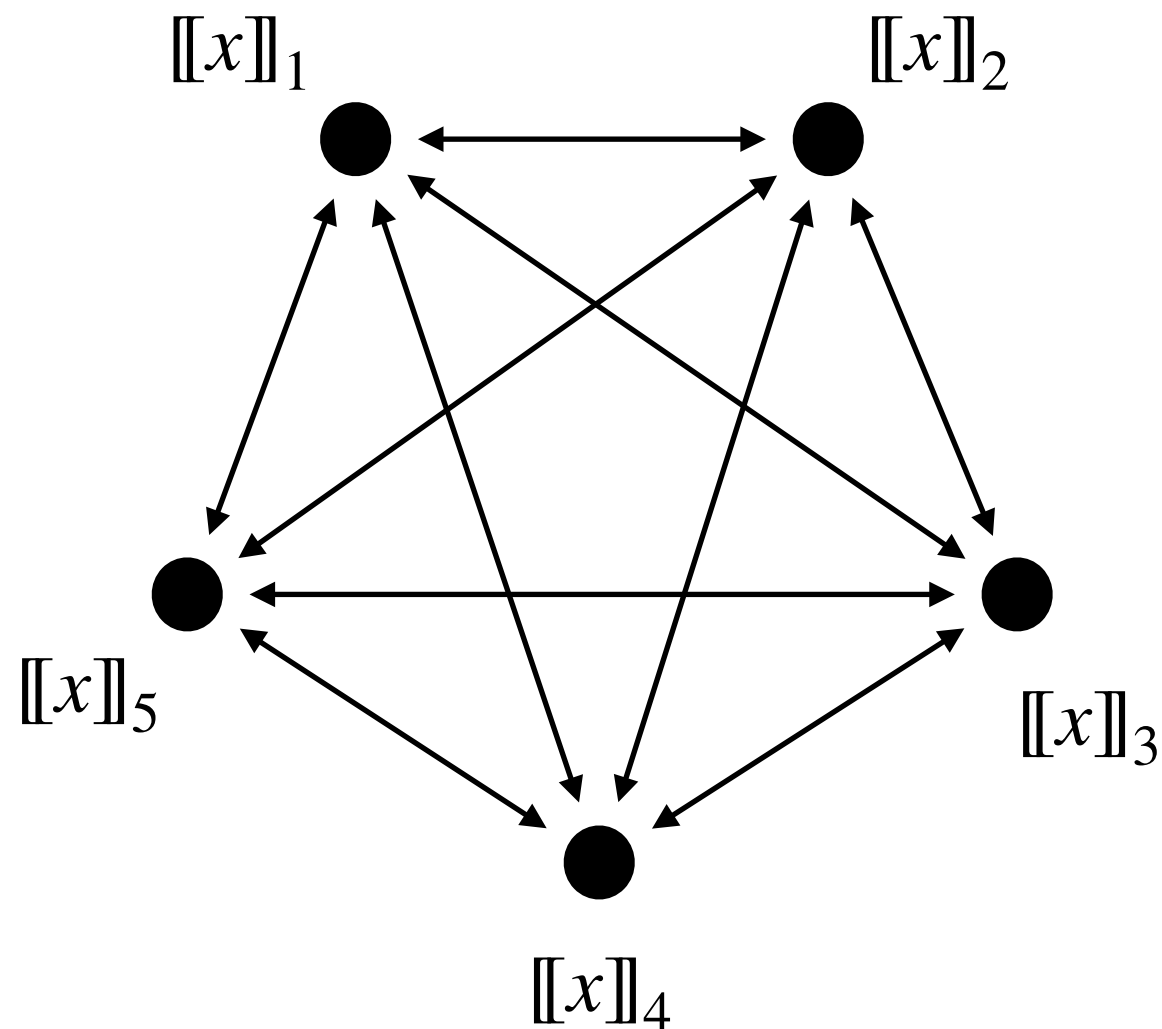
MPC-in-the-Head transform

Zero-knowledge proof



MPCitH: general principle

MPC model



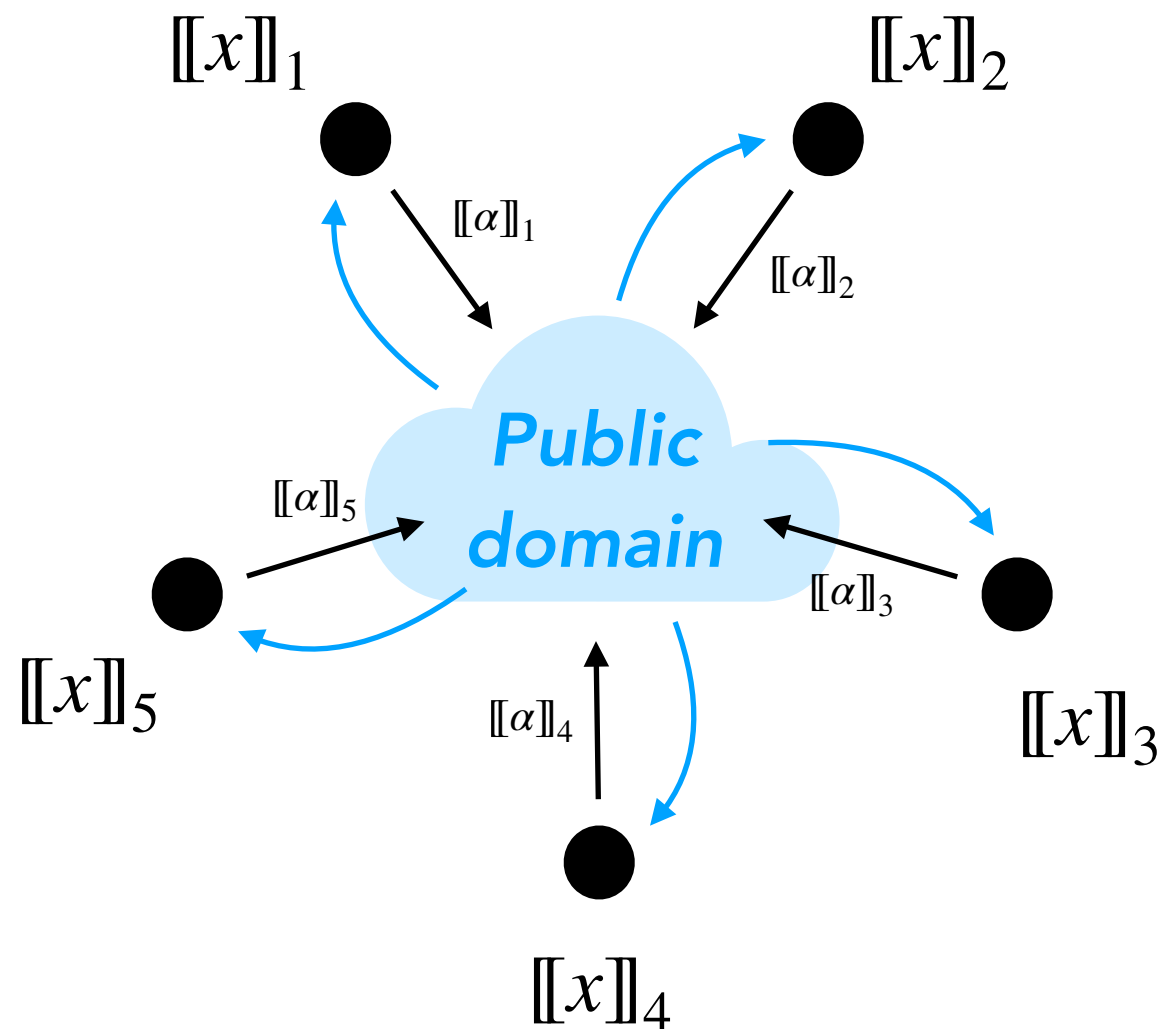
$$x = [[x]]_1 + [[x]]_2 + \dots + [[x]]_N$$

- **Jointly compute**

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

- $(N - 1)$ **private**: the views of any $N - 1$ parties provide no information on x
- **Semi-honest model**: assuming that the parties follow the steps of the protocol

MPC model



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- $(N - 1)$ **private**: the views of any $N - 1$ parties provide no information on x
- **Semi-honest model**: assuming that the parties follow the steps of the protocol
- **Broadcast model**
 - ▶ Parties locally compute on their shares $[[x]] \mapsto [[\alpha]]$
 - ▶ Parties broadcast $[[\alpha]]$ and recompute α
 - ▶ Parties start again (now knowing α)

MPCitH transform

Prover

Verifier

MPCitH transform

- ① Generate and commit shares
 $[[x]] = ([[x]]_1, \dots, [[x]]_N)$

$\text{Com}^{\rho_1}([[x]]_1)$
...
 $\text{Com}^{\rho_N}([[x]]_N)$

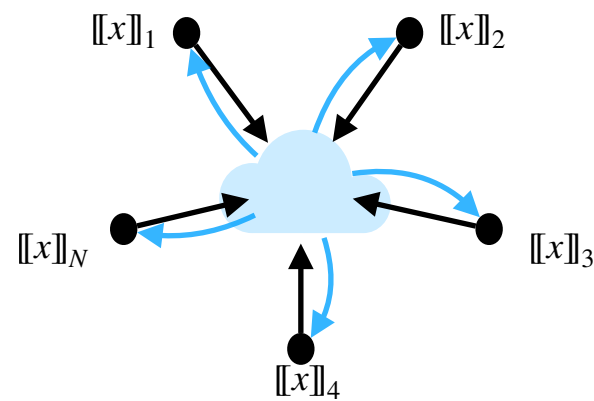
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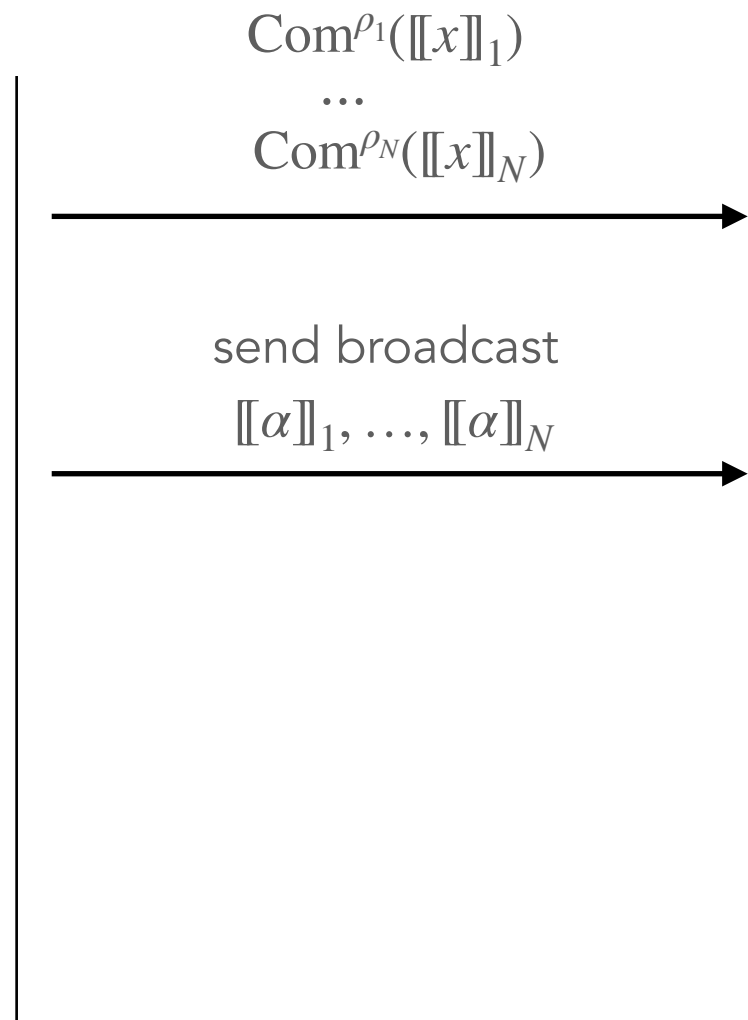
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- ② Run MPC in their head



Prover



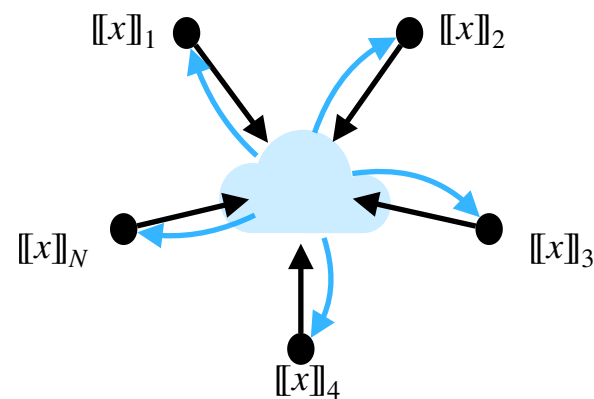
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Prover

$\text{Com}^{\rho_1}([[x]]_1)$

\dots
 $\text{Com}^{\rho_N}([[x]]_N)$

send broadcast

$[[a]]_1, \dots, [[a]]_N$

i^*

- ③ Choose a random party

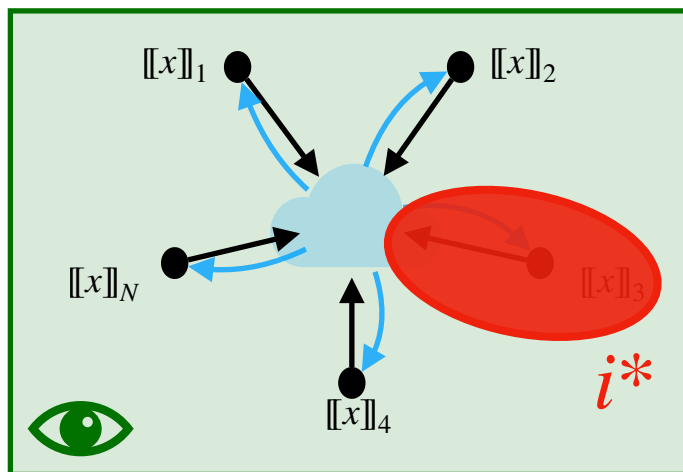
$$i^* \leftarrow^{\$} \{1, \dots, N\}$$

Verifier

MPCitH transform

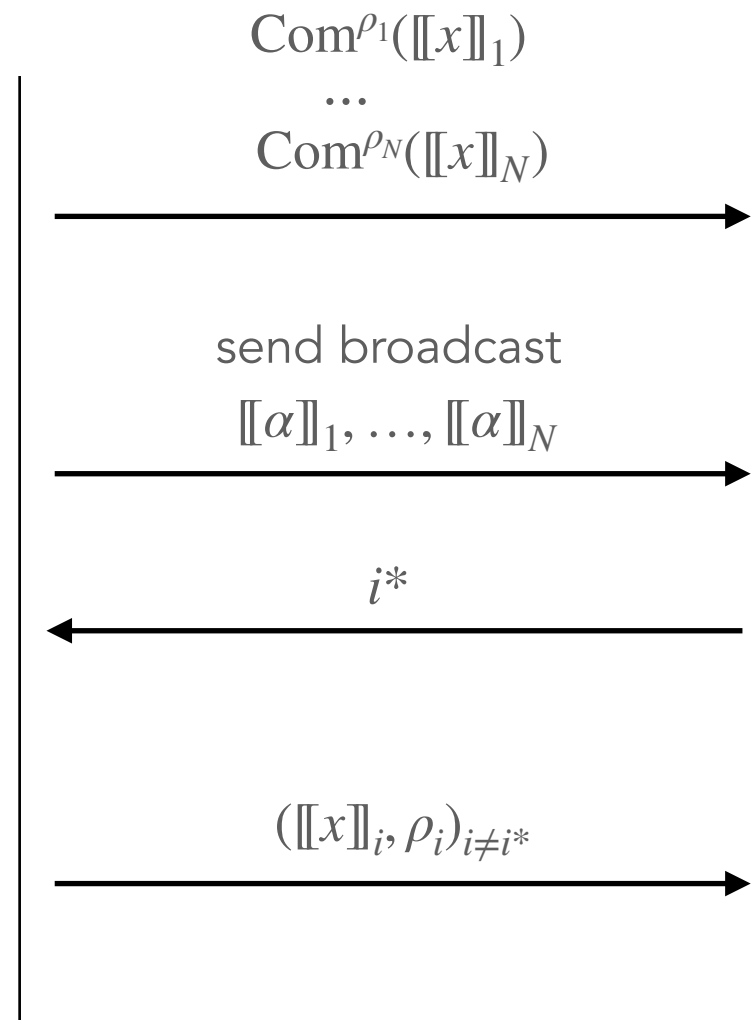
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④ Open parties $\{1, \dots, N\} \setminus \{i^*\}$

Prover



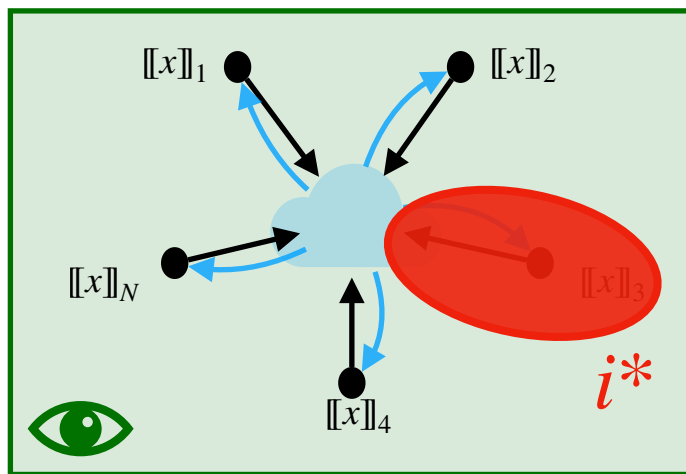
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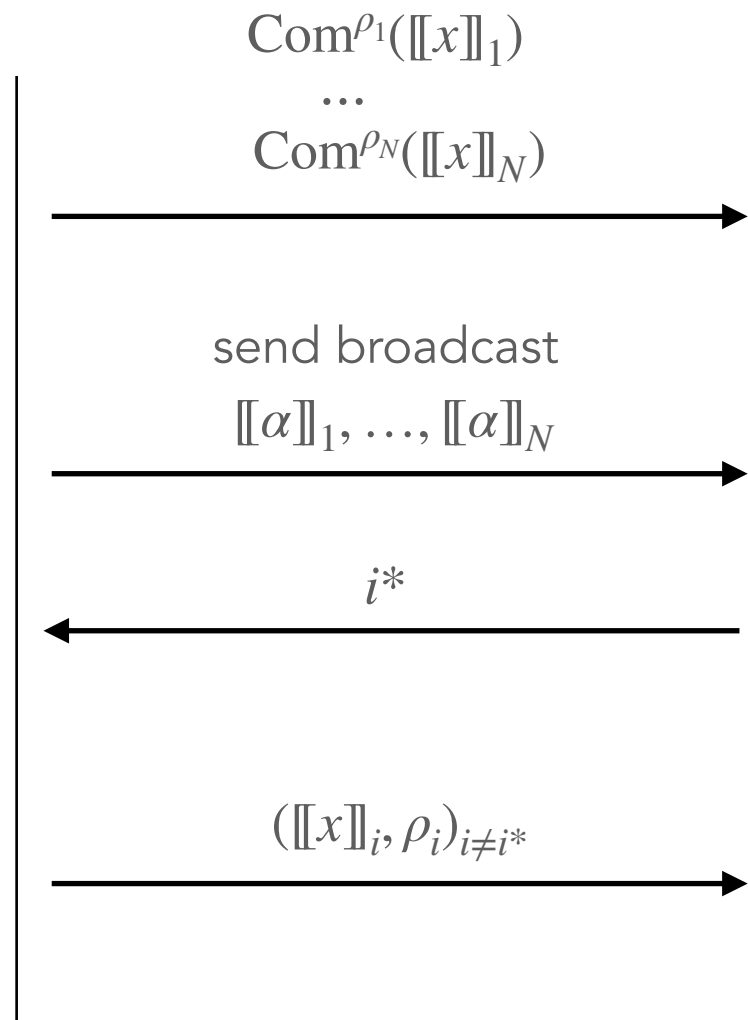
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③ Choose a random party
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⑤ Check $\forall i \neq i^*$
 - Commitments $\text{Com}^{\rho_i}([[x]]_i)$
 - MPC computation $[[\alpha]]_i = \varphi([[x]]_i)$
 Check $g(y, \alpha) = \text{Accept}$

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MPCitH transform

- ① Generate and commit shares

$$[[x]] = ([[x]]_1, \dots, [[x]]_N)$$

We have $F(x) \neq y$ where

$$x := [[x]]_1 + \dots + [[x]]_N$$

$\text{Com}^{\rho_1}([[x]]_1)$

\dots

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Malicious Prover

Verifier

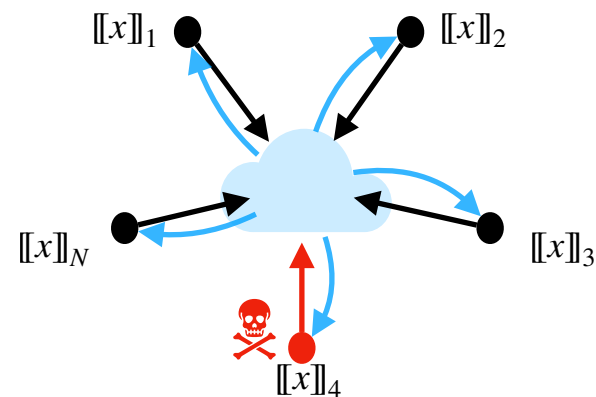
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- ② Run MPC in their head



$\text{Com}^{\rho_1}([[x]]_1)$

...

$\text{Com}^{\rho_N}([[x]]_N)$

send broadcast

$[[\alpha]]_1, \dots, [[\alpha]]_N$

Malicious Prover

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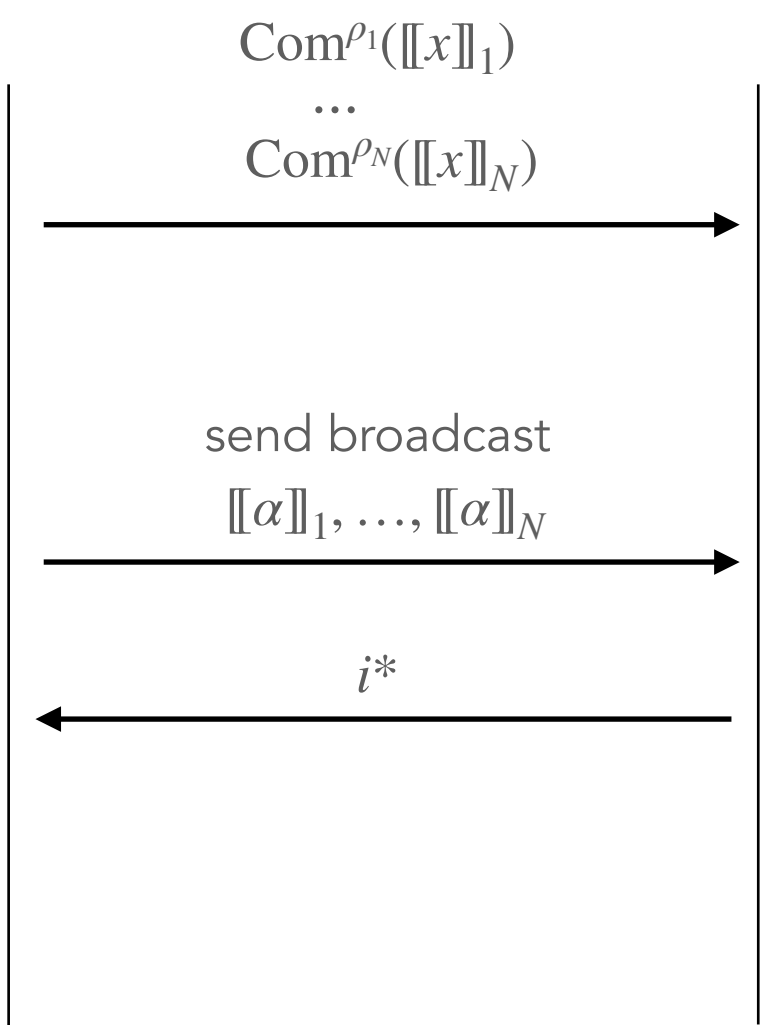
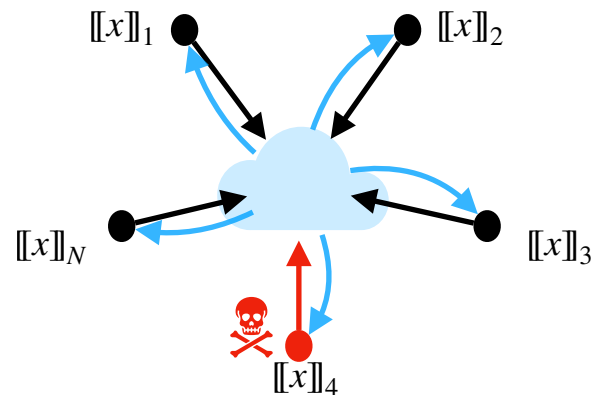
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$$i^* \leftarrow^{\$} \{1, \dots, N\}$$

Malicious Prover

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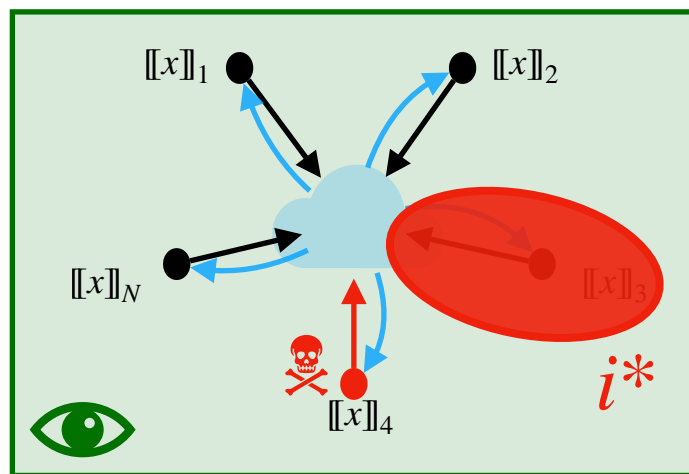
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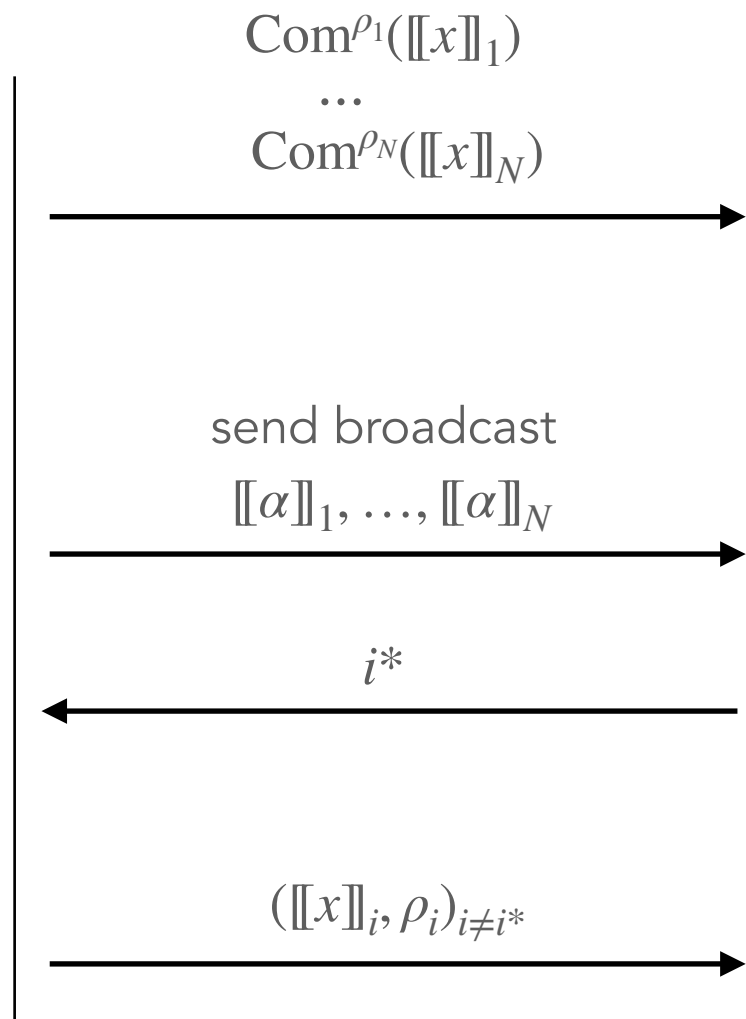
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- ④ Open parties $\{1, \dots, N\} \setminus \{i^*\}$



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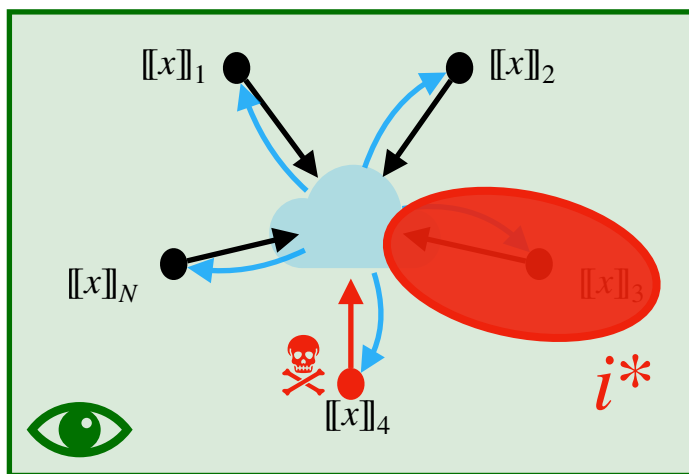
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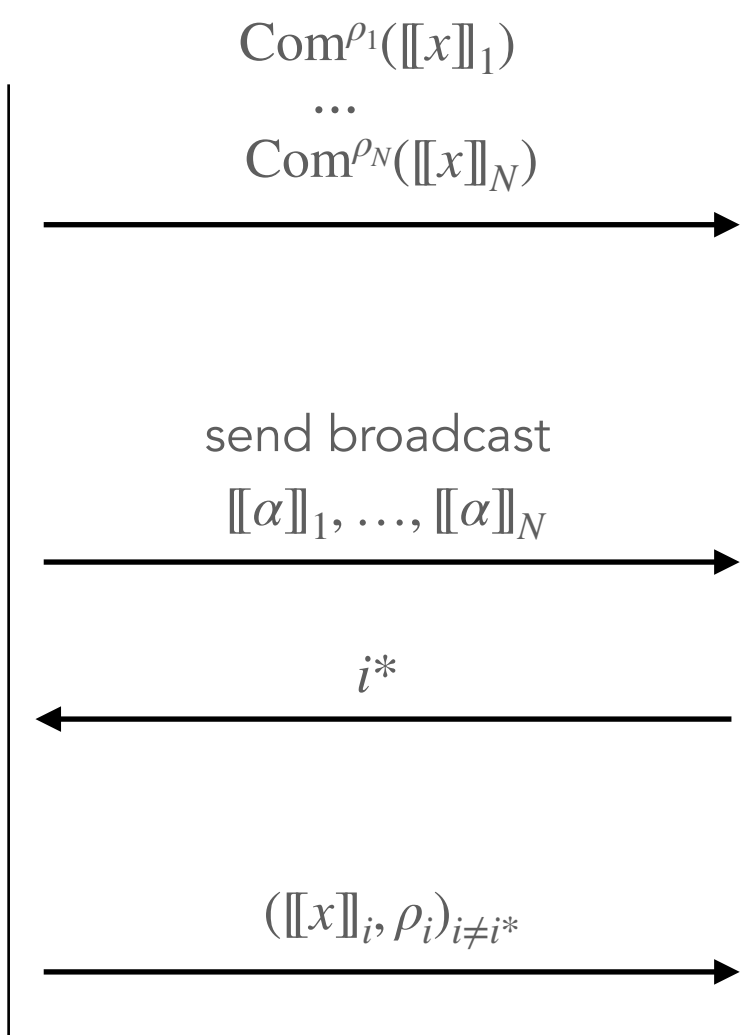
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Malicious Prover

Verifier

✗ Cheating detected!

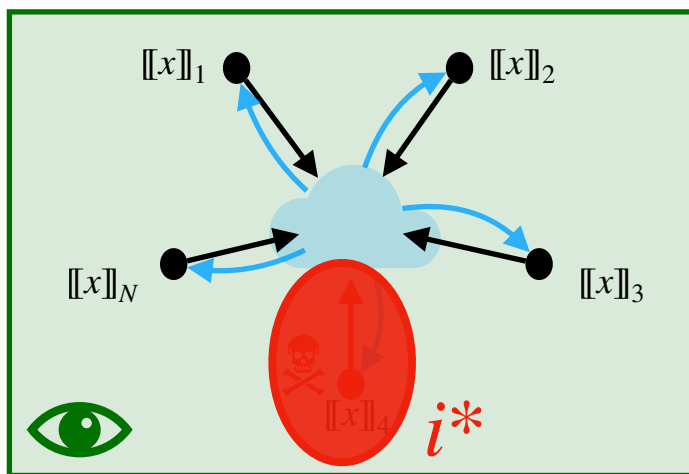
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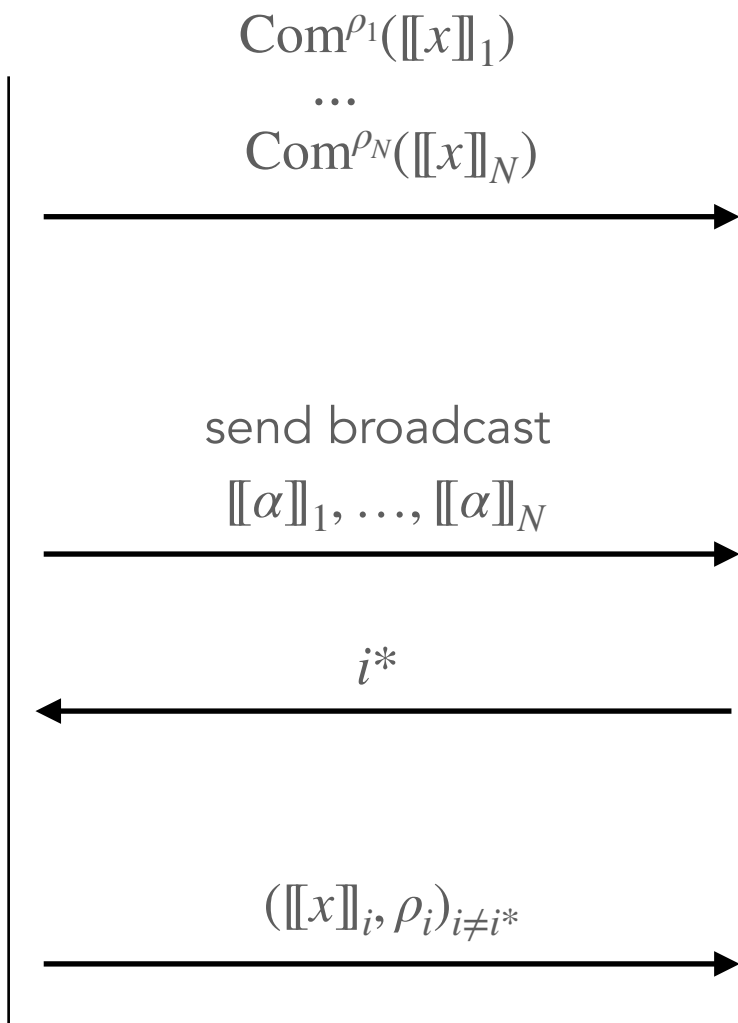
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Seems OK.

MPCitH transform

- Zero-knowledge \iff MPC protocol is $(N - 1)$ -private

MPCitH transform

- **Zero-knowledge** \iff MPC protocol is $(N - 1)$ -private
- **Soundness:**

$$\begin{aligned} & \mathbb{P}(\text{malicious prover convinces the verifier}) \\ &= \mathbb{P}(\text{corrupted party remains hidden}) \\ &= \frac{1}{N} \end{aligned}$$

MPCitH transform

- **Zero-knowledge** \iff MPC protocol is $(N - 1)$ -private
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- **Parallel repetition**

Protocol repeated τ times in parallel \rightarrow soundness error $\left(\frac{1}{N}\right)^\tau$

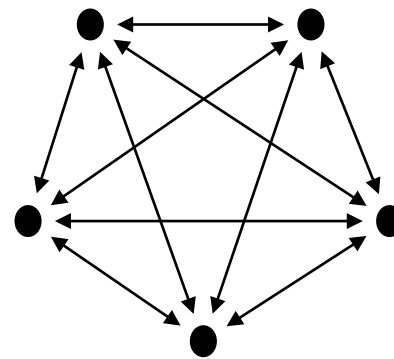
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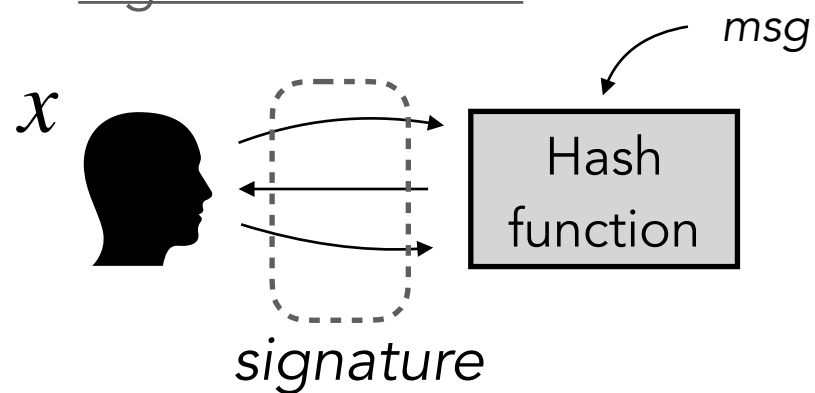
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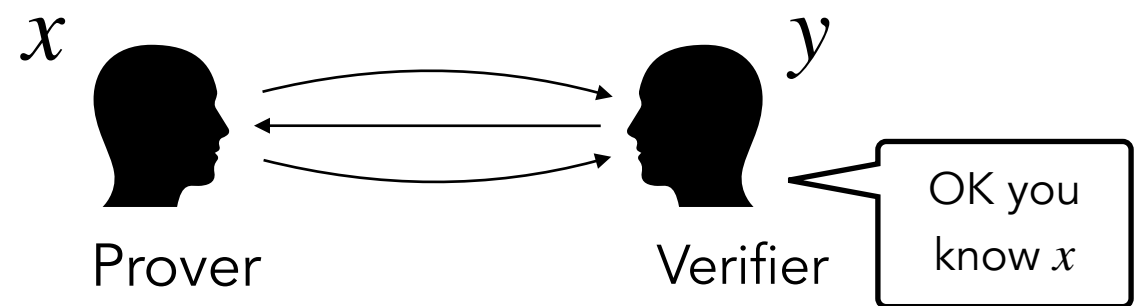
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Signature scheme



Zero-knowledge proof

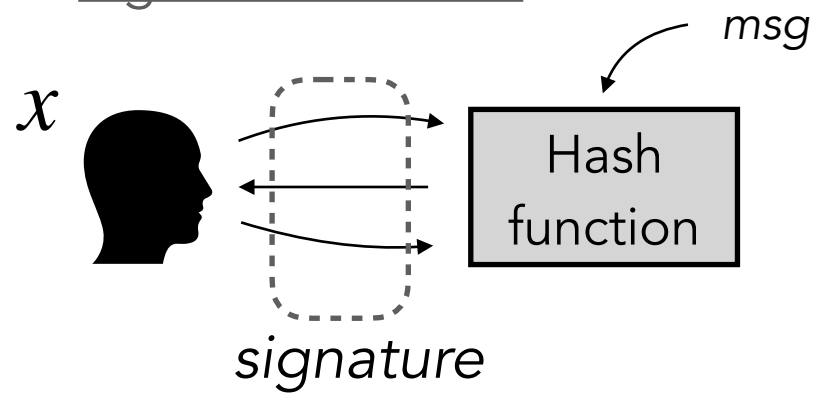


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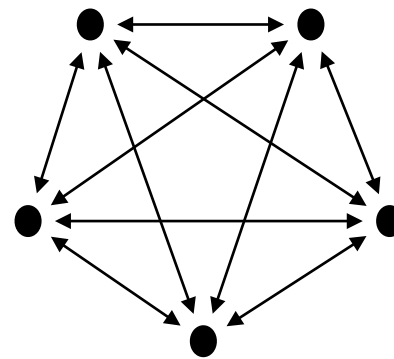
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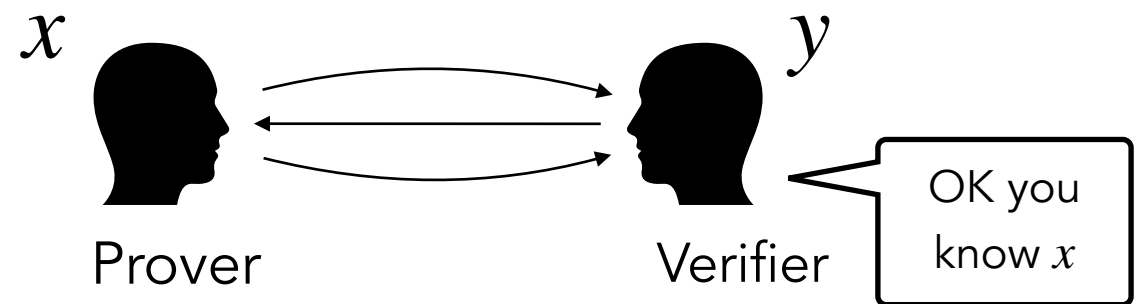


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MPC-in-the Head transform

Zero-knowledge proof



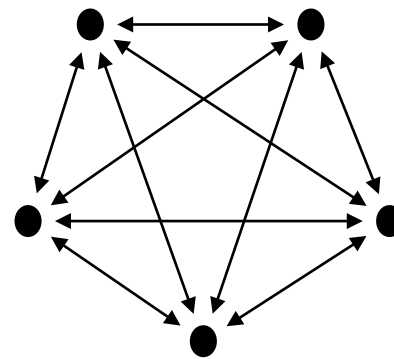


One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

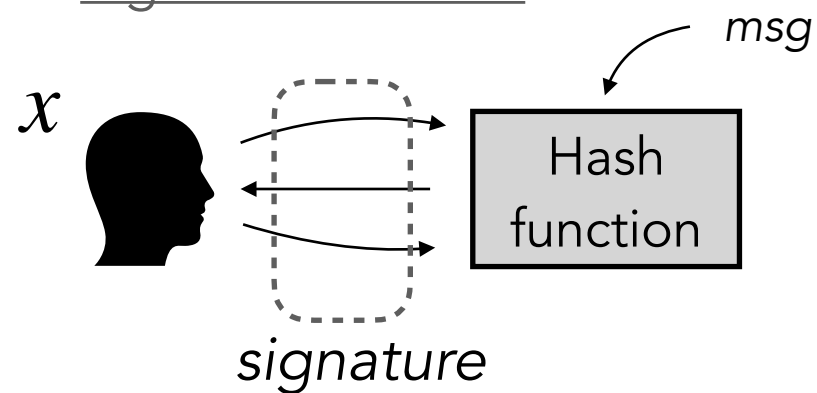
Multiparty computation (MPC)



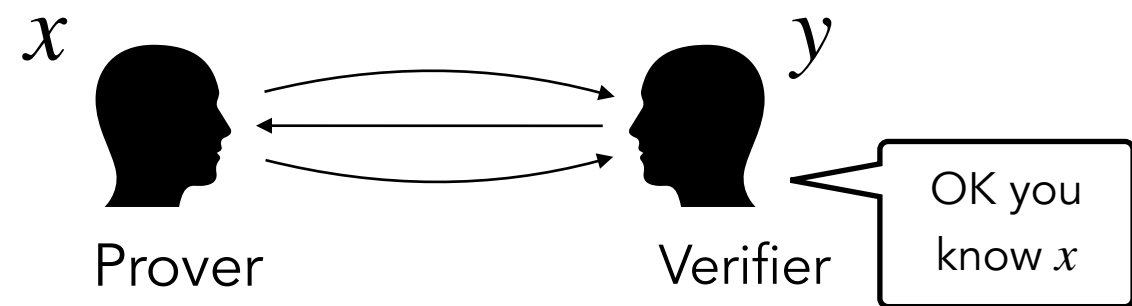
Input sharing $[[x]]$
Joint evaluation of:

$$g(x) = \begin{cases} \text{Accept} & \text{if } F(x) = y \\ \text{Reject} & \text{if } F(x) \neq y \end{cases}$$

Signature scheme



Zero-knowledge proof



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One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
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Three approaches:

■ Rely on standard symmetric primitives

- AES: *BBQ* (2019), *Banquet* (2021), *Limbo-Sign* (2021), *Helium+AES* (2022)

One-way function

$$F : x \mapsto y$$

E.g. AES, MQ system,
Syndrome decoding

Three approaches:

- Rely on standard symmetric primitives
- Rely on MPC-friendly symmetric primitives
 - LowMC: *Picnic1* (2017), *Picnic2* (2018), *Picnic3* (2020)
 - Rain: *Rainier* (2021), *BN++Rain* (2022)
 - AIM: *AIMer* (2022)

One-way function

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E.g. AES, MQ system,
Syndrome decoding

Three approaches:

- Rely on standard symmetric primitives
- Rely on MPC-friendly symmetric primitives
- Rely on well-known hard problems (*non-exhaustive list*)
 - Syndrome Decoding: *SDitH* (2022), *RYDE* (2023)
 - MinRank: *MiRitH* (2022), *MIRA* (2023)
 - Multivariate Quadratic: *MQOM* (2023), *Biscuit* (2023)
 - Permuted Kernel: *PERK* (2023)

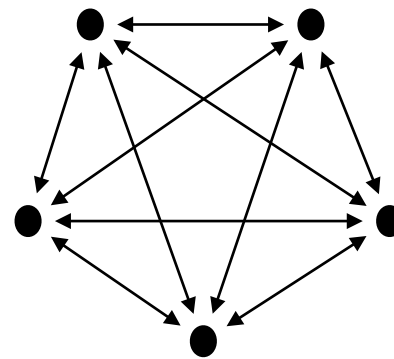
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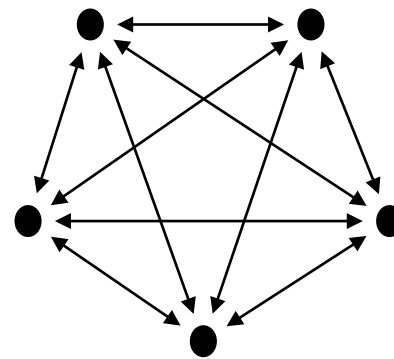
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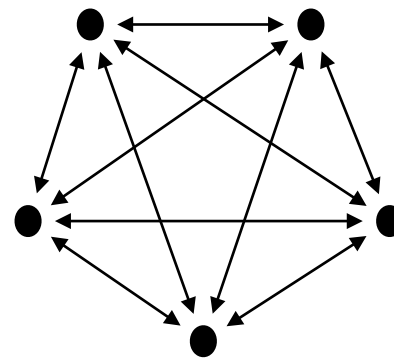
Expressed as an arithmetic circuit, enabling us to use existing MPCitH-based proof systems (as BN++)

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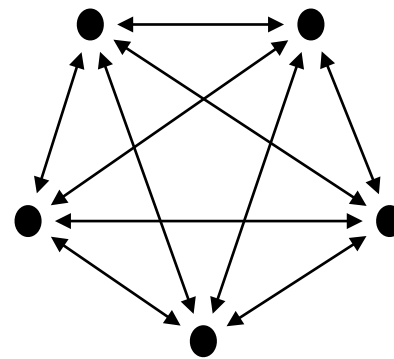
Should be rephrased to achieve interesting performances

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Should be rephrased to achieve interesting performances

Example (RYDE): how to check that a vector $x \in \mathbb{F}_{q^m}^n$ has a rank weight smaller than some public bound r ?

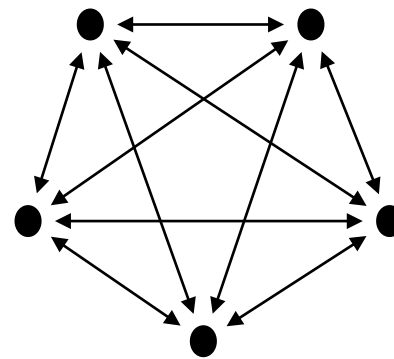
By checking that x_1, \dots, x_n are roots of a degree- q^r q -polynomial $\sum_{i=0}^r a_i X^{q^i}$.

One-way function

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E.g. AES, MQ system,
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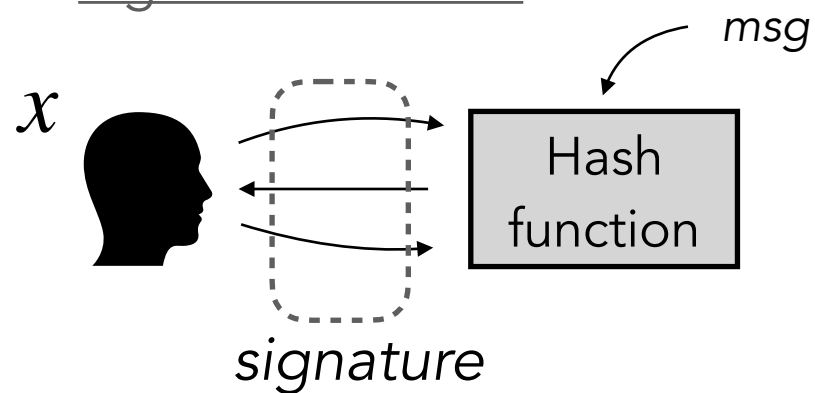
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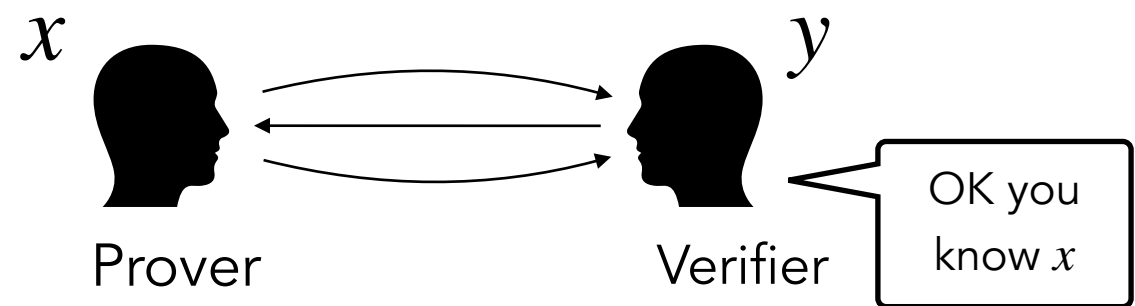
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Signature scheme



Zero-knowledge proof

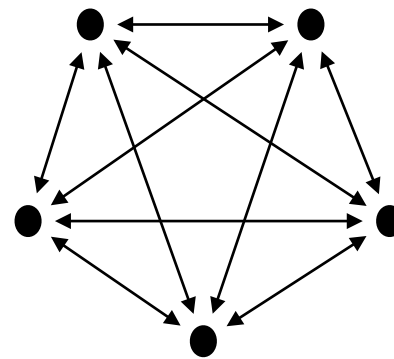


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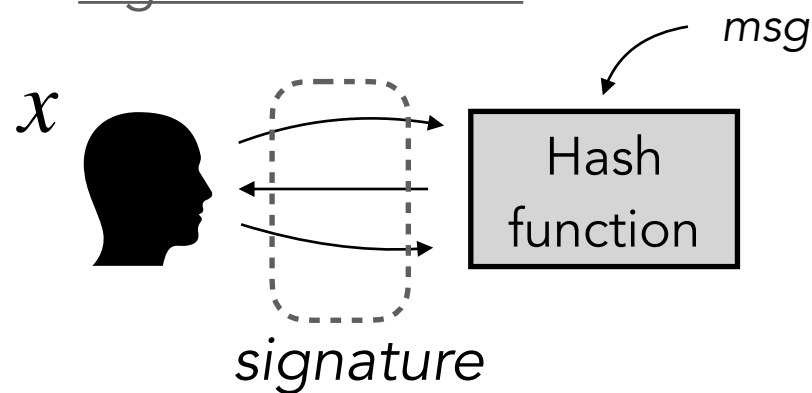
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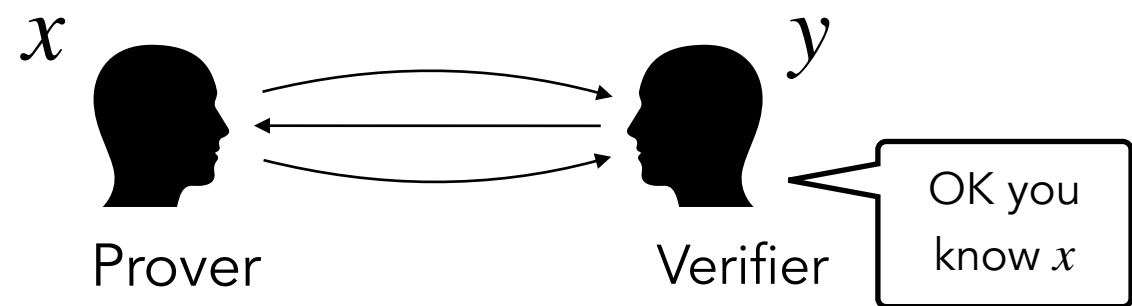
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Signature scheme



Zero-knowledge proof



Fiat-Shamir transform

Should take [KZ20] attack into account (when there are more than 3 rounds)!

[KZ20] Kales, Zaverucha. "An attack on some signature schemes constructed from five-pass identification schemes" (CANS20)

Optimisations and variants

Optimisations and variants

With `SDitH-L1-gf251` as example.

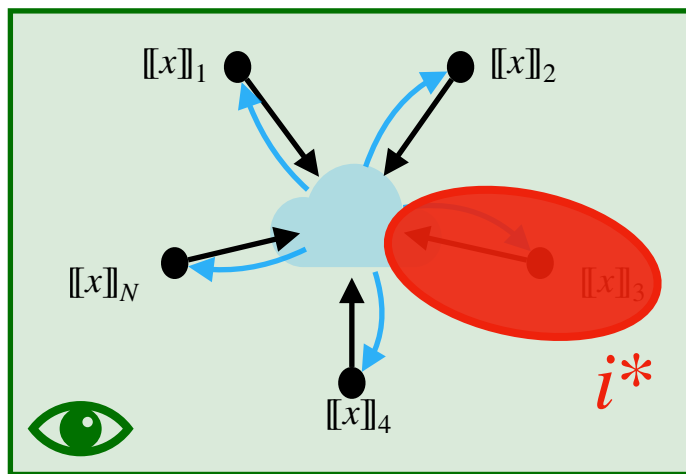
Field $GF(251)$

NIST Category I

MPCitH transform

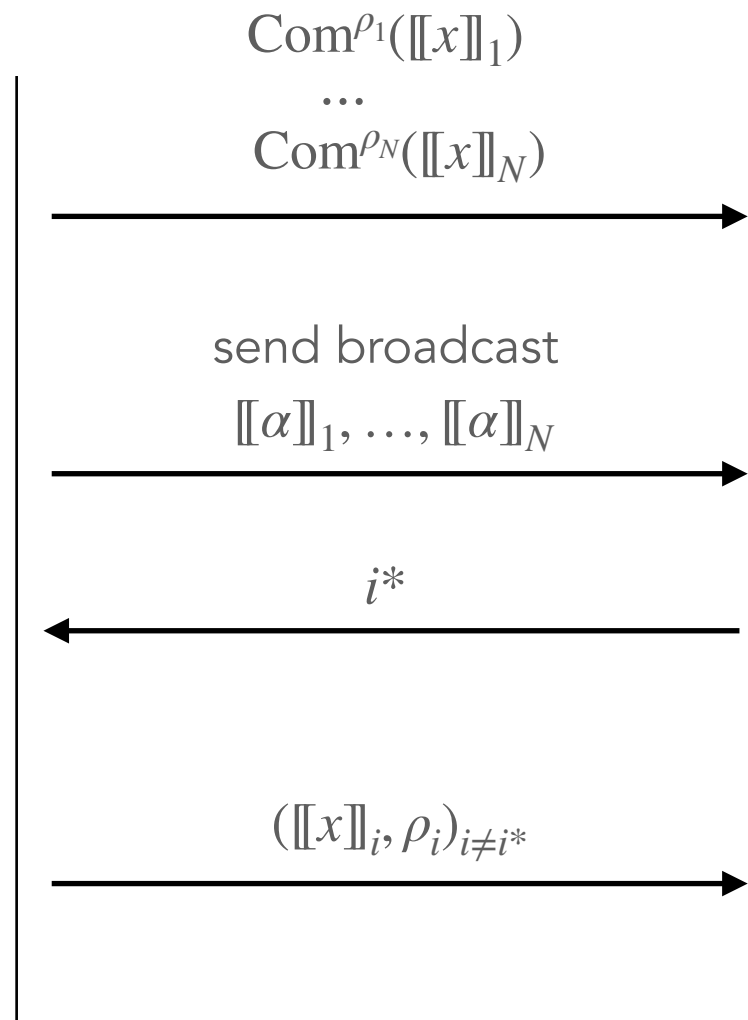
① Generate and commit shares
 $[[x]] = ([[x]]_1, \dots, [[x]]_N)$

② Run MPC in their head



④ Open parties $\{1, \dots, N\} \setminus \{i^*\}$

Prover

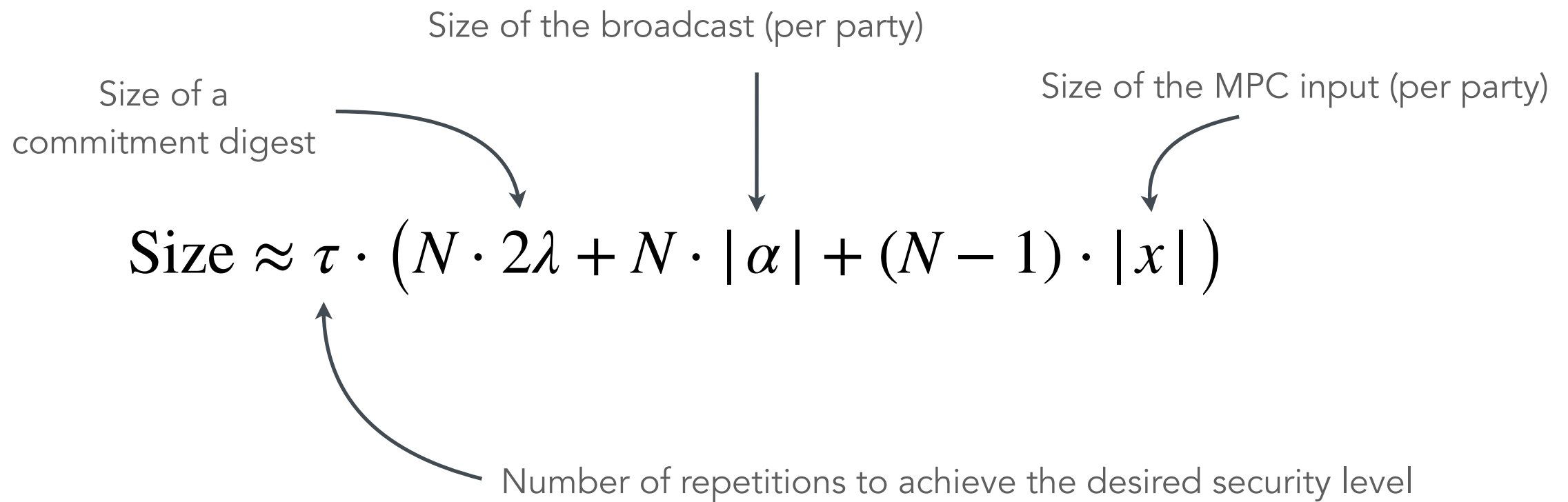


③ Choose a random party
 $i^* \leftarrow^{\$} \{1, \dots, N\}$

⑤ Check $\forall i \neq i^*$
 - Commitments $\text{Com}^{\rho_i}([[x]]_i)$
 - MPC computation $[[\alpha]]_i = \varphi([[x]]_i)$
 Check $g(y, \alpha) = \text{Accept}$

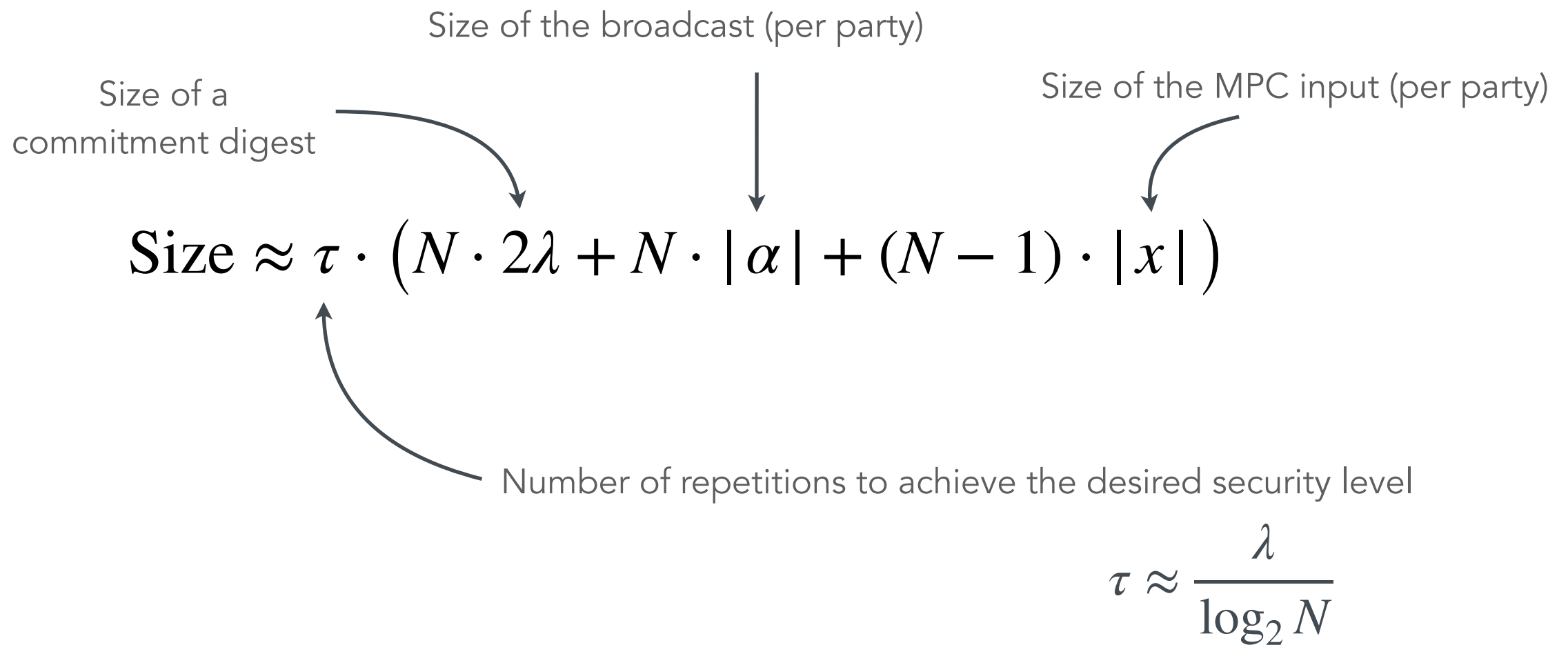
Verifier

Naive MPCitH transformation



$$\tau \approx \frac{\lambda}{\log_2 N}$$

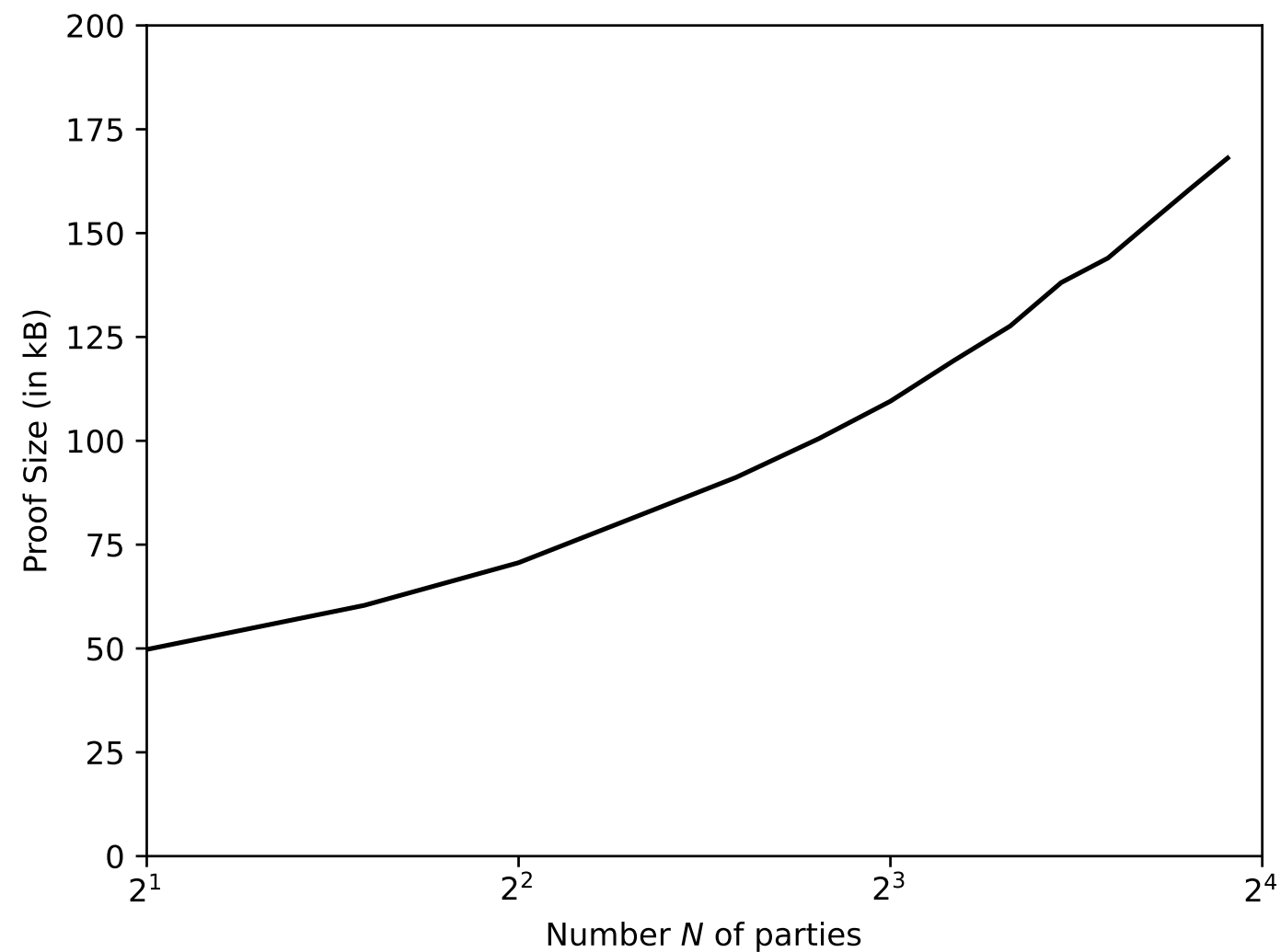
Naive MPCitH transformation



SDitH-L1-gf251:

the input x of the MPC protocol is around **323** bytes,
The broadcast value α of the MPC protocol is around **36** bytes.

Naive MPCitH transformation



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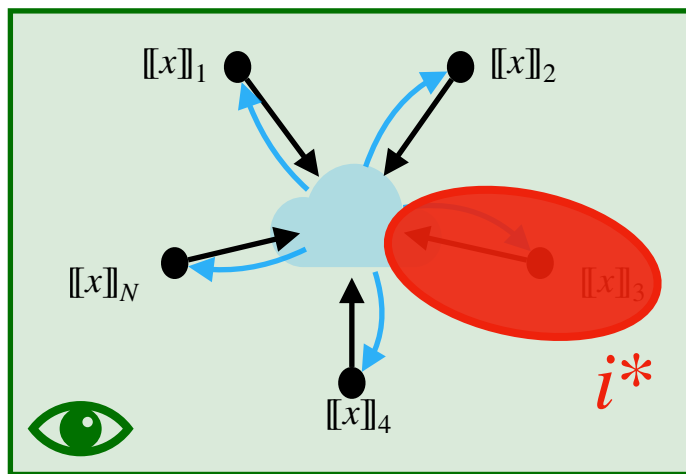
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MPCitH transform

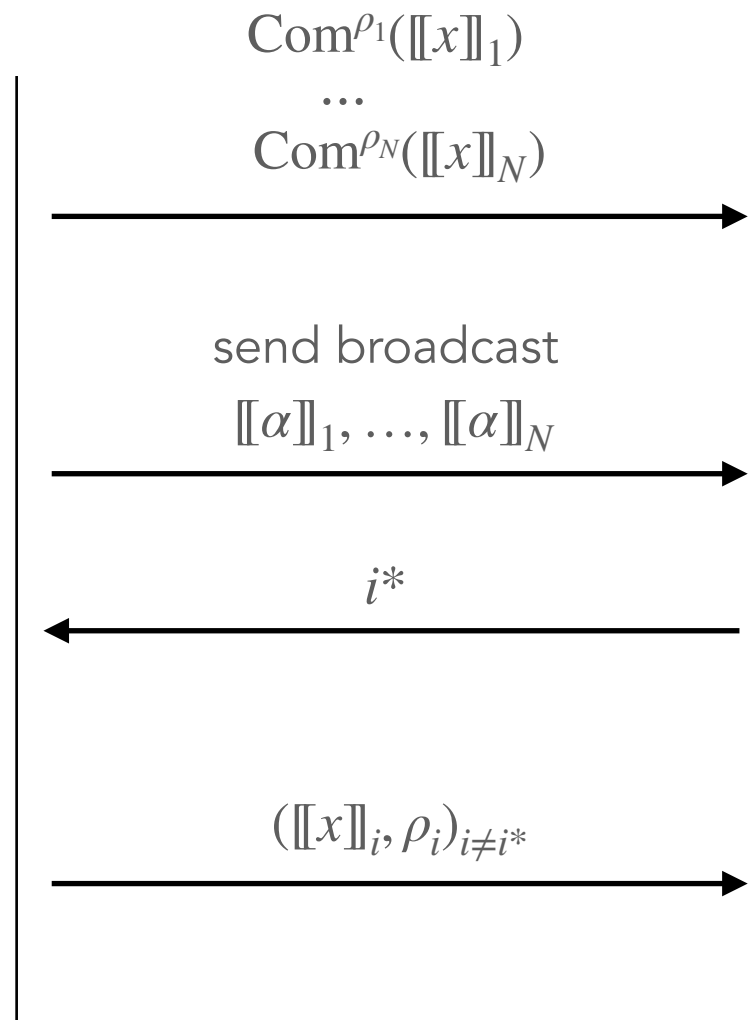
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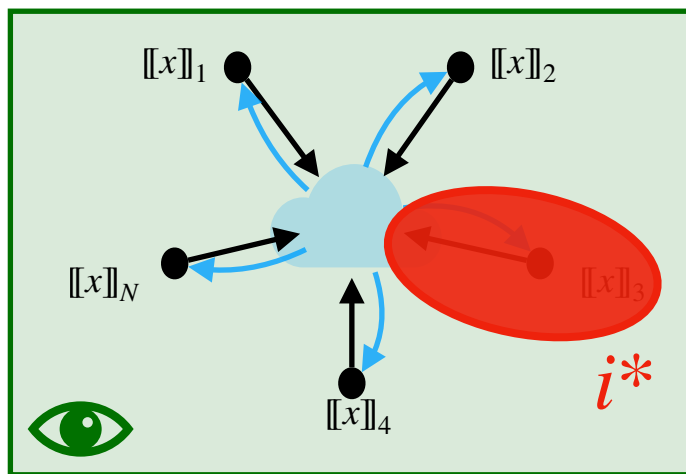
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$$[[x]] = ([[x]]_1, \dots, [[x]]_N)$$

Compute

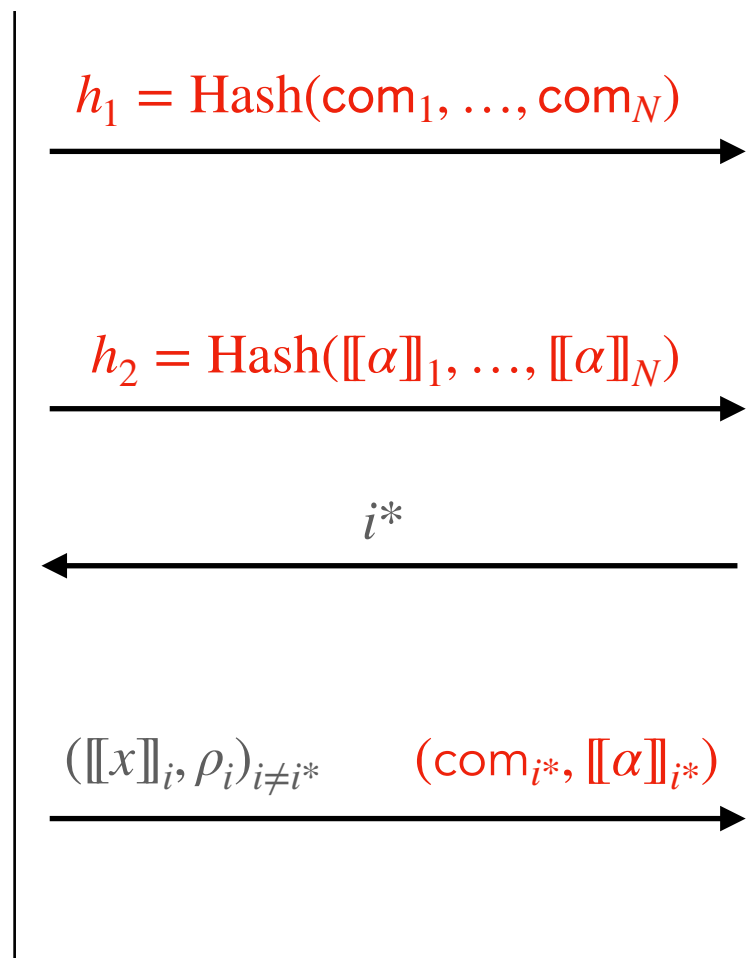
$$\forall i, \text{com}_i = \text{Com}^{\rho_i}([[x]]_i)$$

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Check $g(y, \alpha) = \text{Accept}$

Check $h_1 = \text{Hash}(\text{com}_1, \dots, \text{com}_N)$

Check $h_2 = \text{Hash}([[alpha]]_1, \dots, [[alpha]]_N)$

Verifier

MPCitH transform

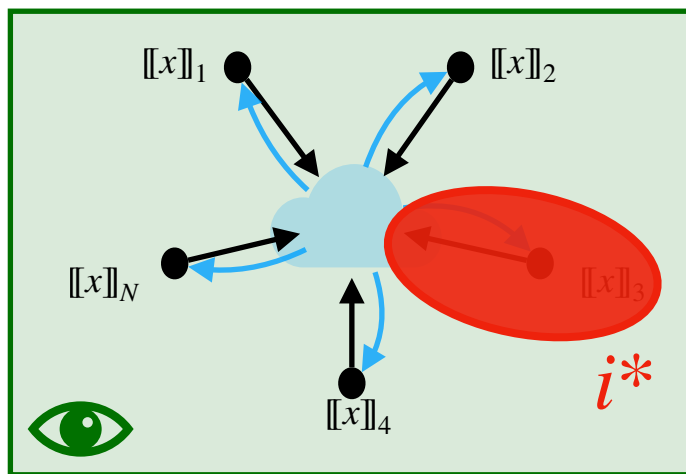
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$$h_1 = \text{Hash}(\text{com}_1, \dots, \text{com}_N)$$

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i^*

$$([[x]]_i, \rho_i)_{i \neq i^*} \quad (\text{com}_{i^*}, [[α]]_{i^*})$$

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Verifier

Using a Seed Tree

[KKW18] Katz, Kolesnikov, Wang: "Improved Non-Interactive Zero Knowledge with Applications to Post-Quantum Signatures" (CCS 2018)

$$x = \llbracket x \rrbracket_1 + \llbracket x \rrbracket_2 + \llbracket x \rrbracket_3 + \dots + \llbracket x \rrbracket_{N-1} + \llbracket x \rrbracket_N$$

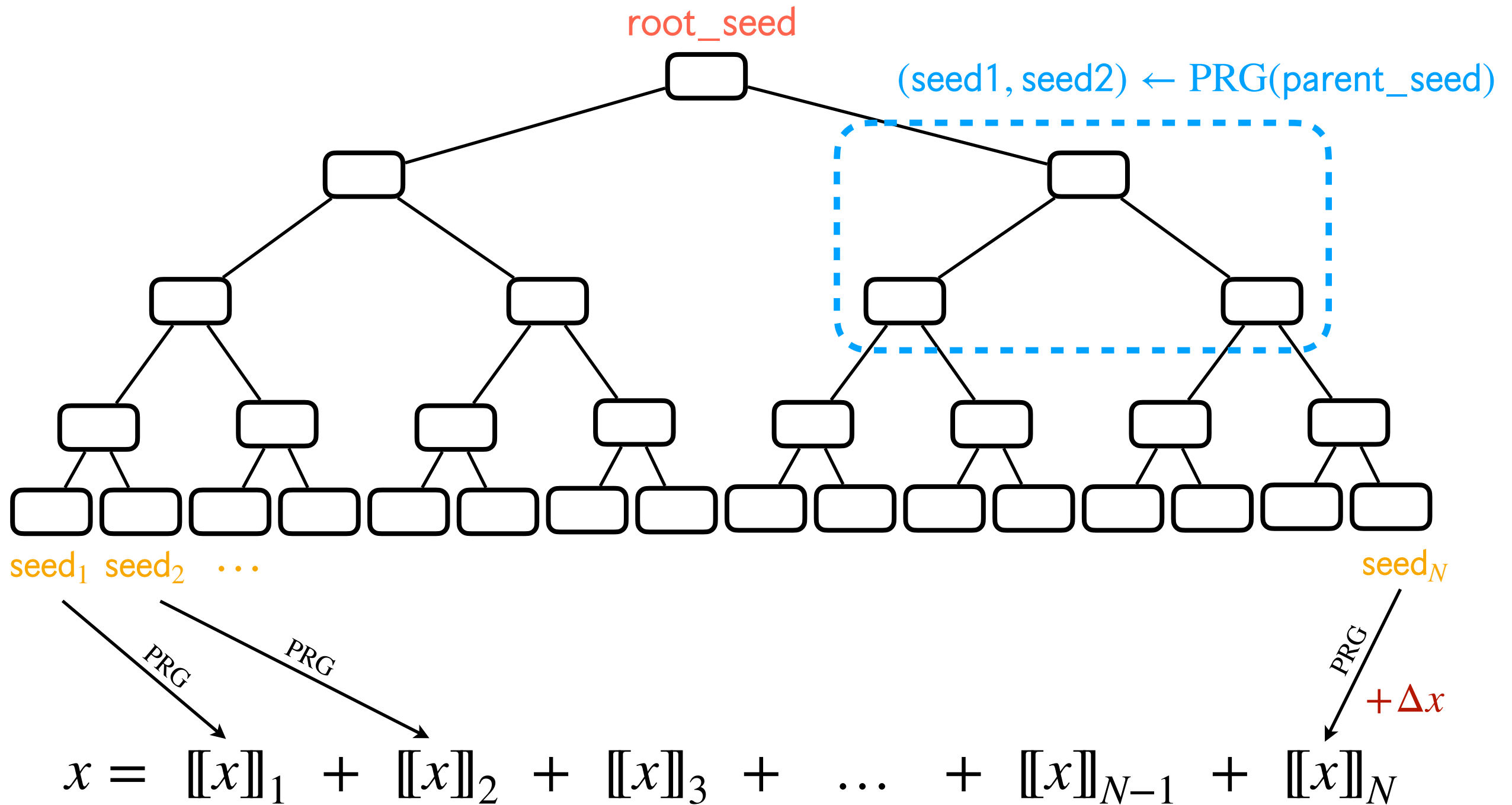
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$$x = \begin{array}{ccccccccc} & \text{seed}_1 & & \text{seed}_2 & & \text{seed}_3 & & & & \text{seed}_{N-1} & & \text{seed}_N \\ & \downarrow \text{PRG} & & \downarrow \text{PRG} & & \downarrow \text{PRG} & & & & \downarrow \text{PRG} & & \downarrow \text{PRG} + \Delta x \\ x = & \llbracket x \rrbracket_1 & + & \llbracket x \rrbracket_2 & + & \llbracket x \rrbracket_3 & + & \dots & + & \llbracket x \rrbracket_{N-1} & + & \llbracket x \rrbracket_N \end{array}$$

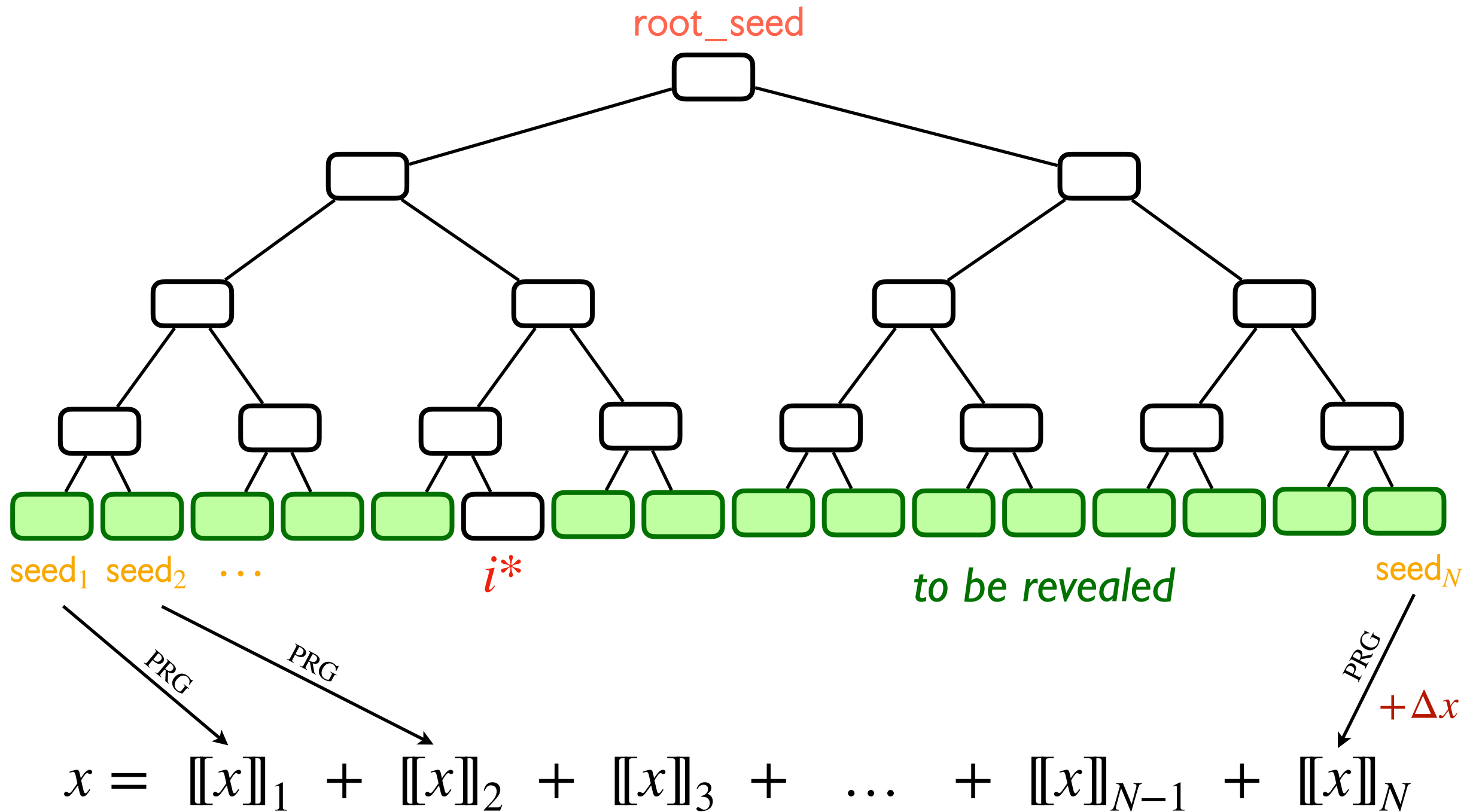
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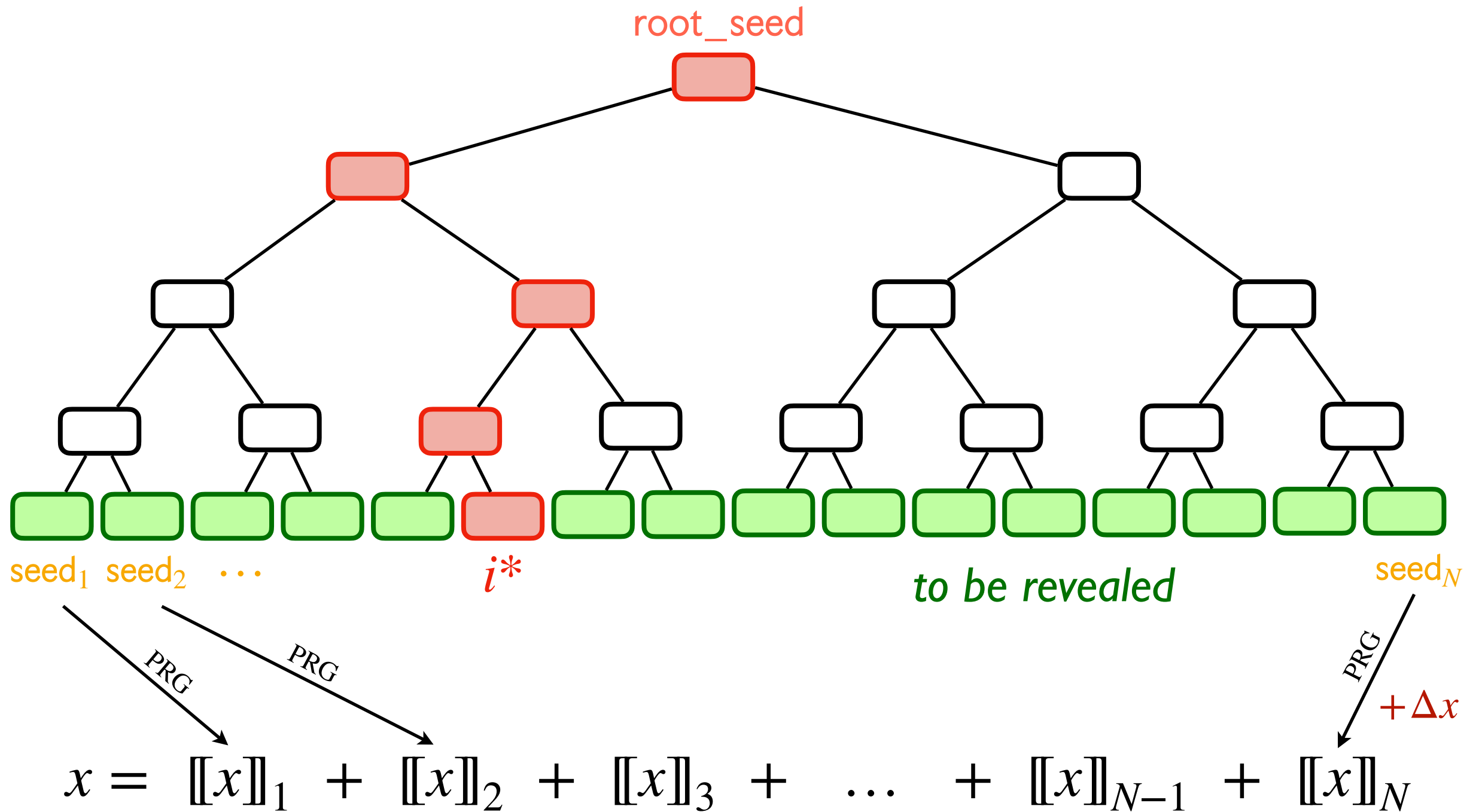
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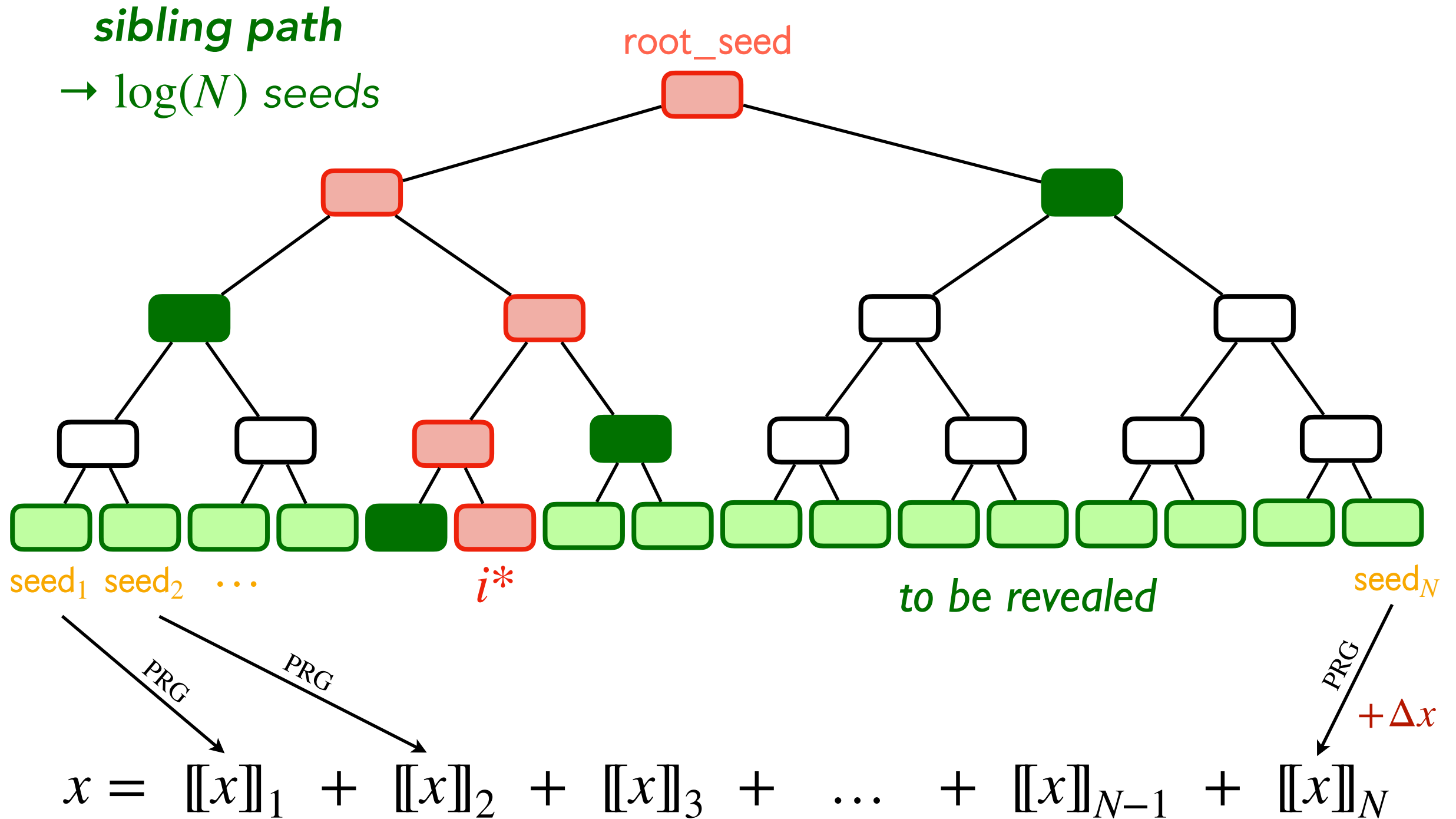
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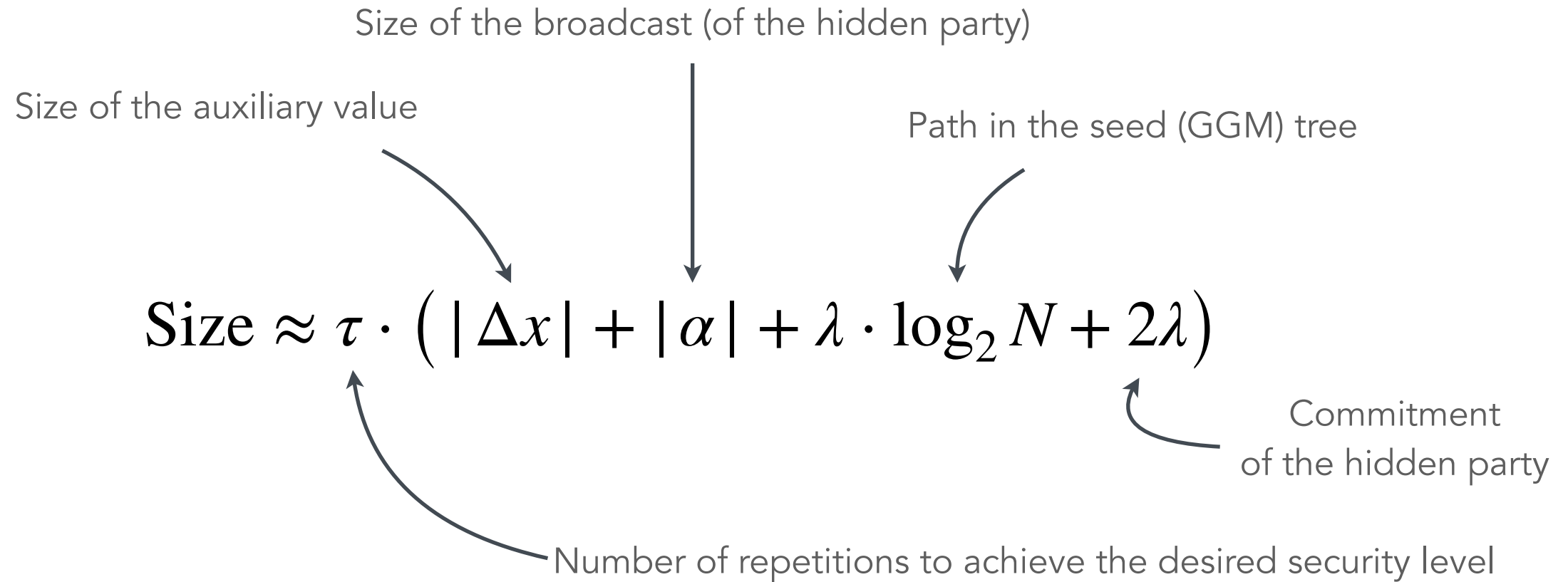


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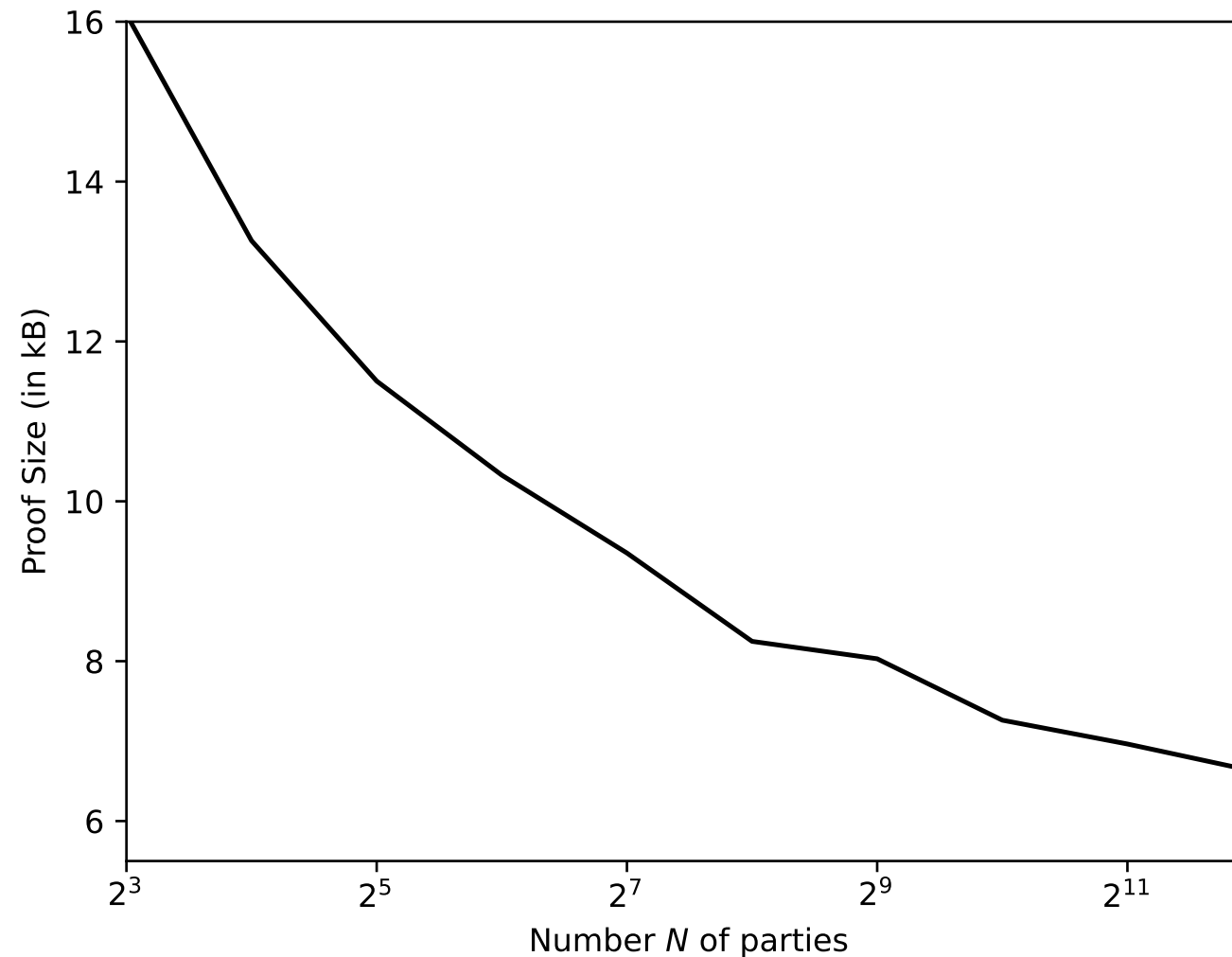


Traditional MPCitH transformation



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Traditional MPCitH transformation



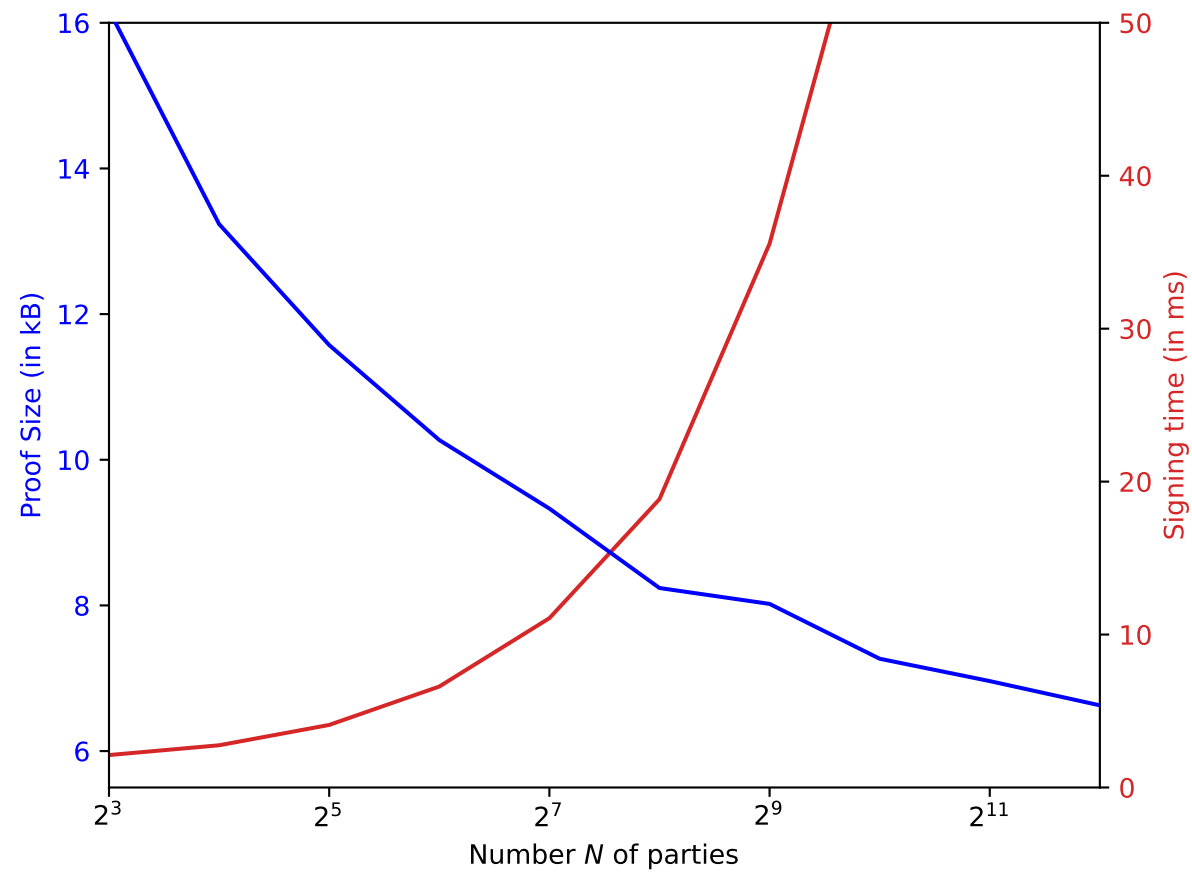
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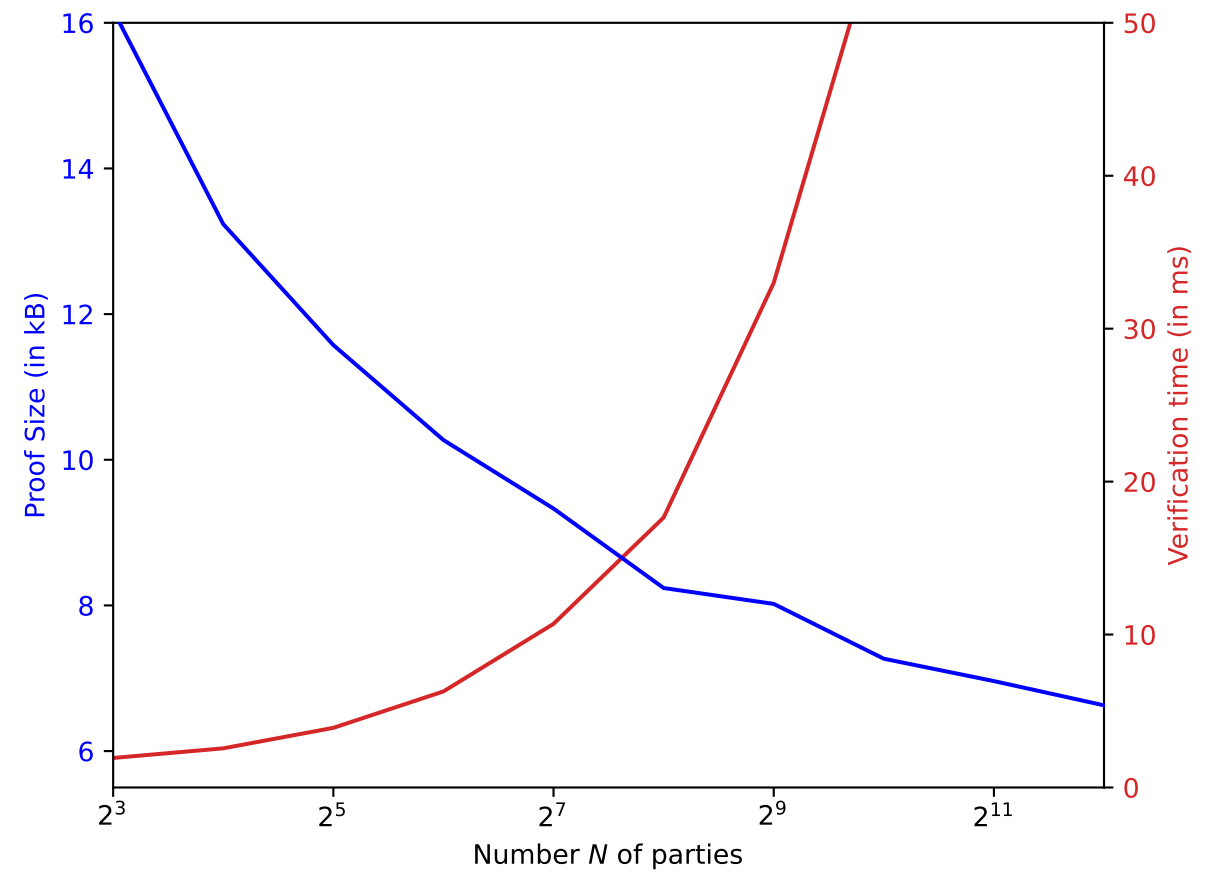
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Traditional MPCitH transformation

Signing algorithm



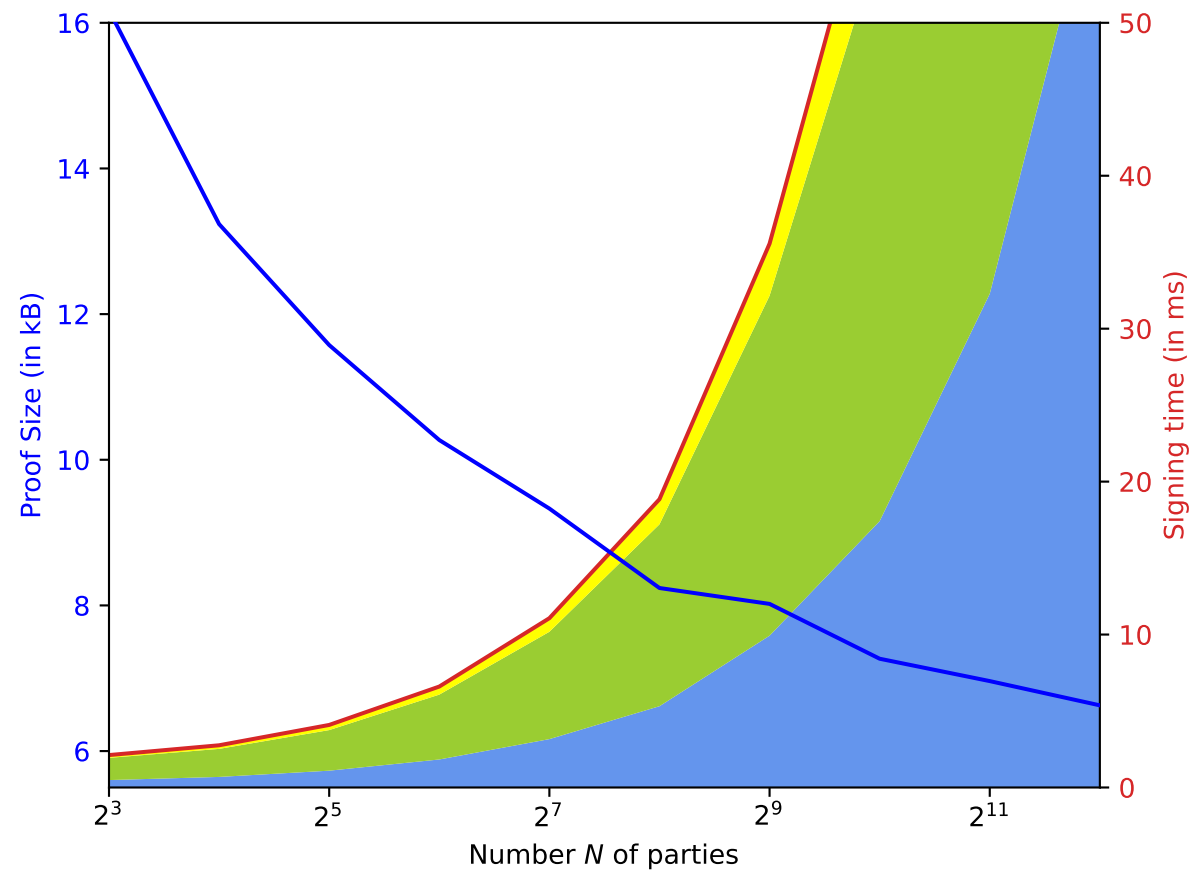
Verification algorithm



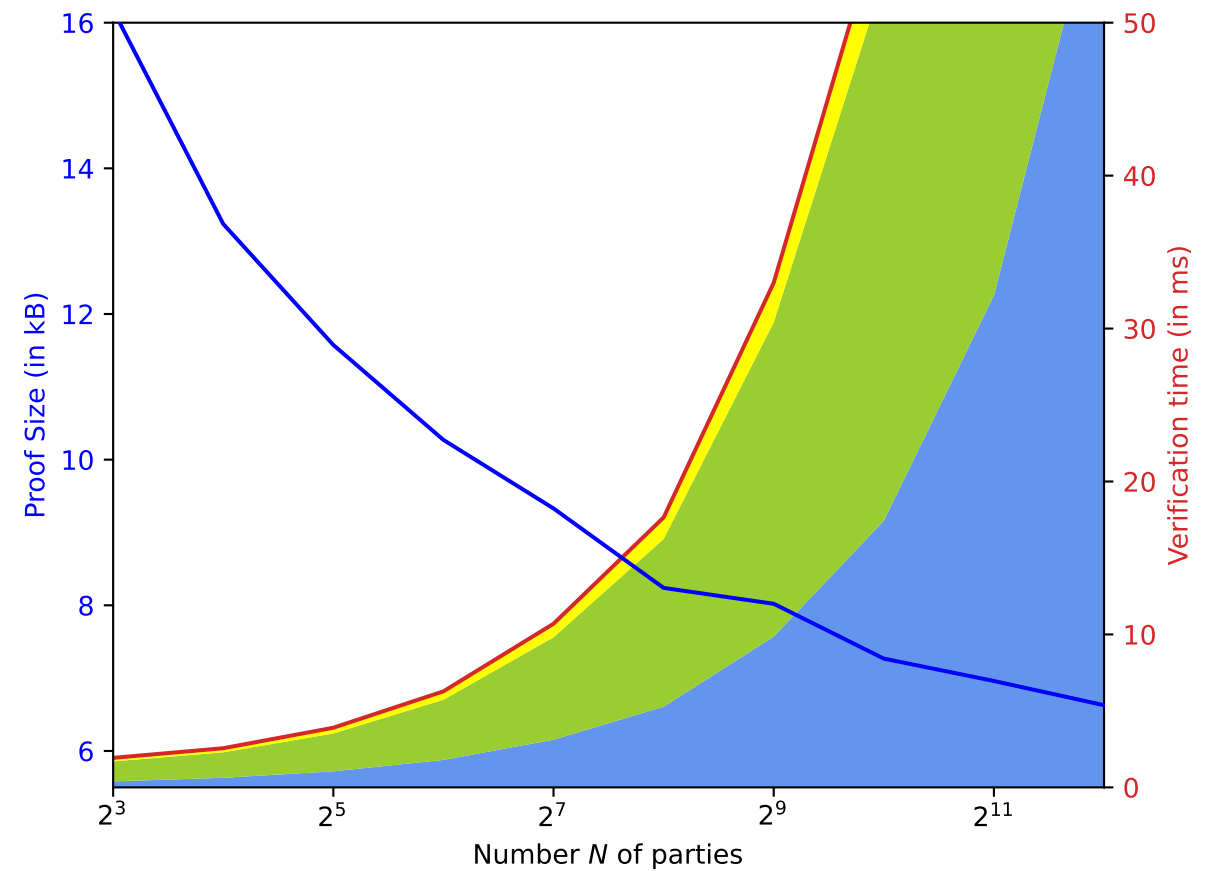
Running times @3.80Ghz

Traditional MPCitH transformation

Signing algorithm



Verification algorithm

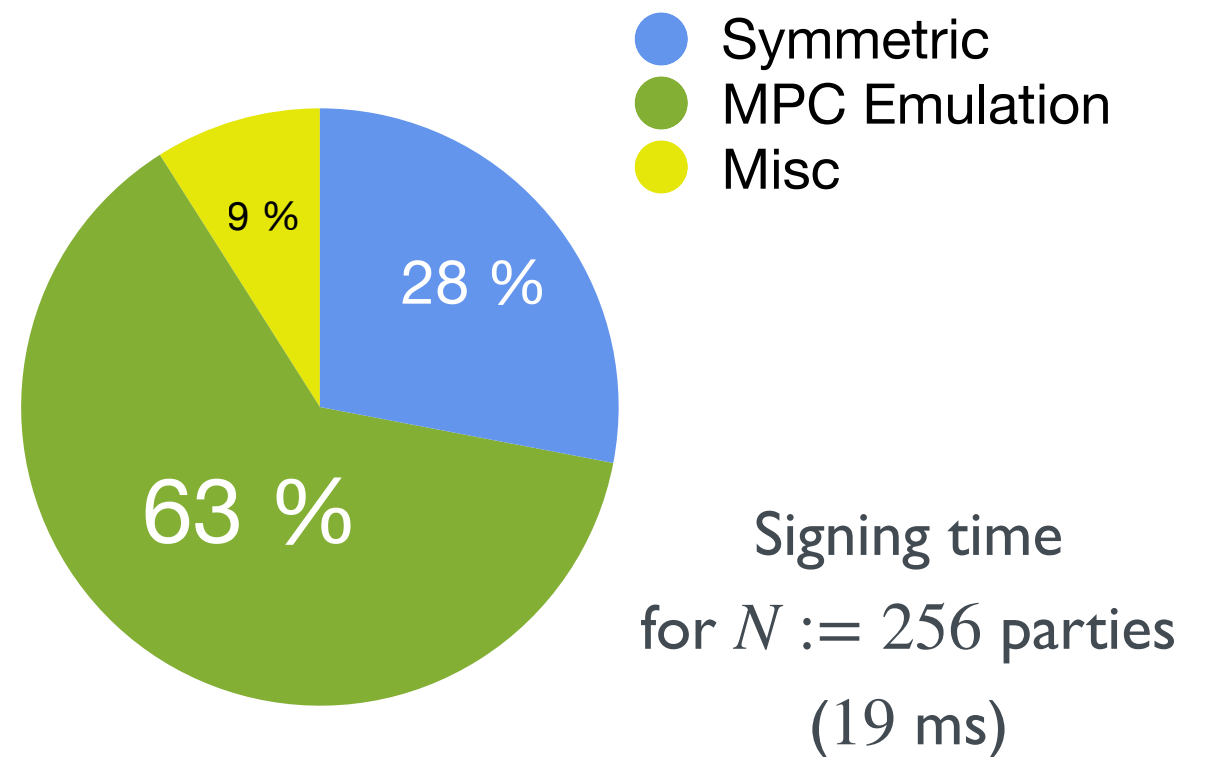
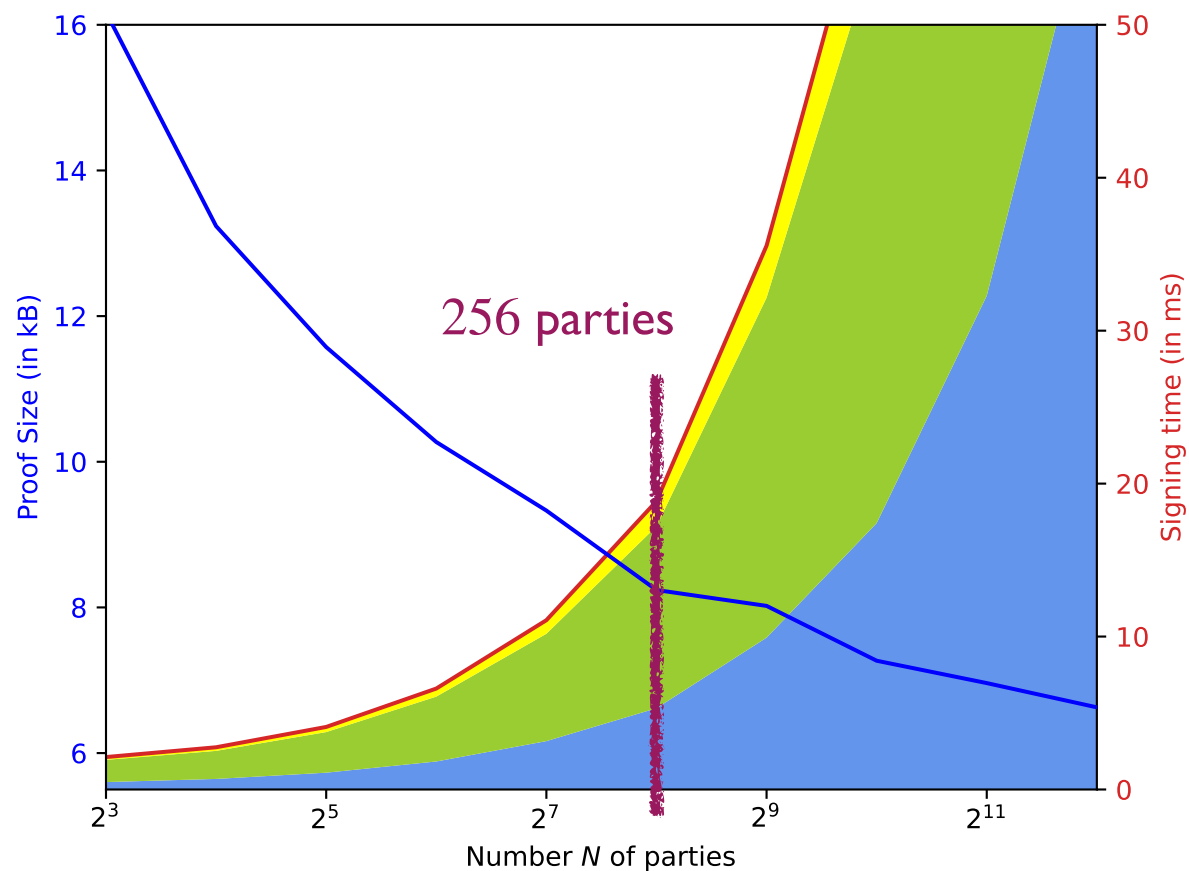


- Symmetric
- MPC Emulation
- Misc

Running times @3.80Ghz

Traditional MPCitH transformation

Signing algorithm



Running times @3.80Ghz

The Hypercube Technique

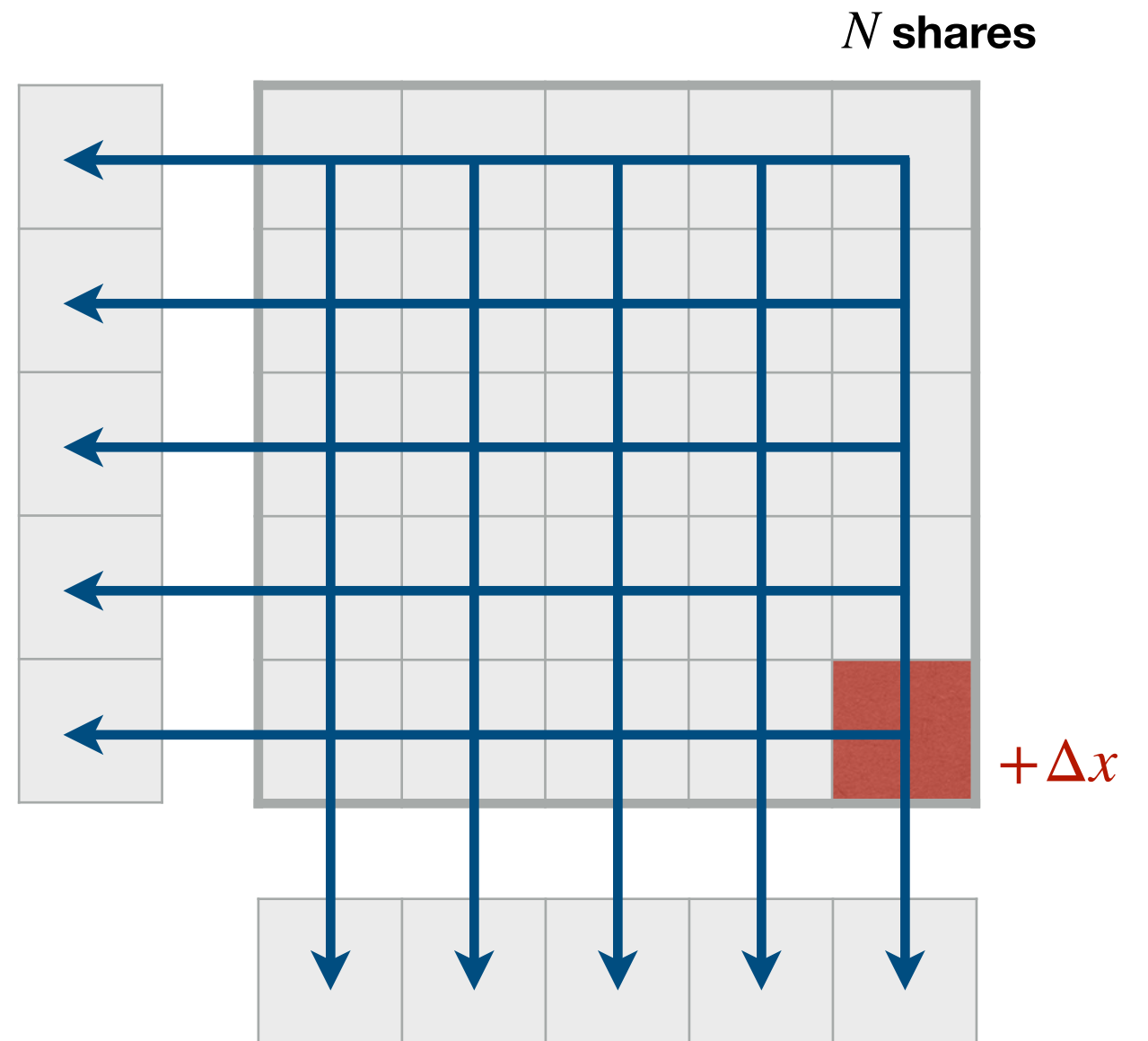
[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: "The Return of the SDitH"
(Eurocrypt 2023)

N shares

$[[x]]_1$	$[[x]]_2$			$[[x]]_{\sqrt{N}}$
$[[x]]_{\sqrt{N}+1}$				
				$[[x]]_N + \Delta x$

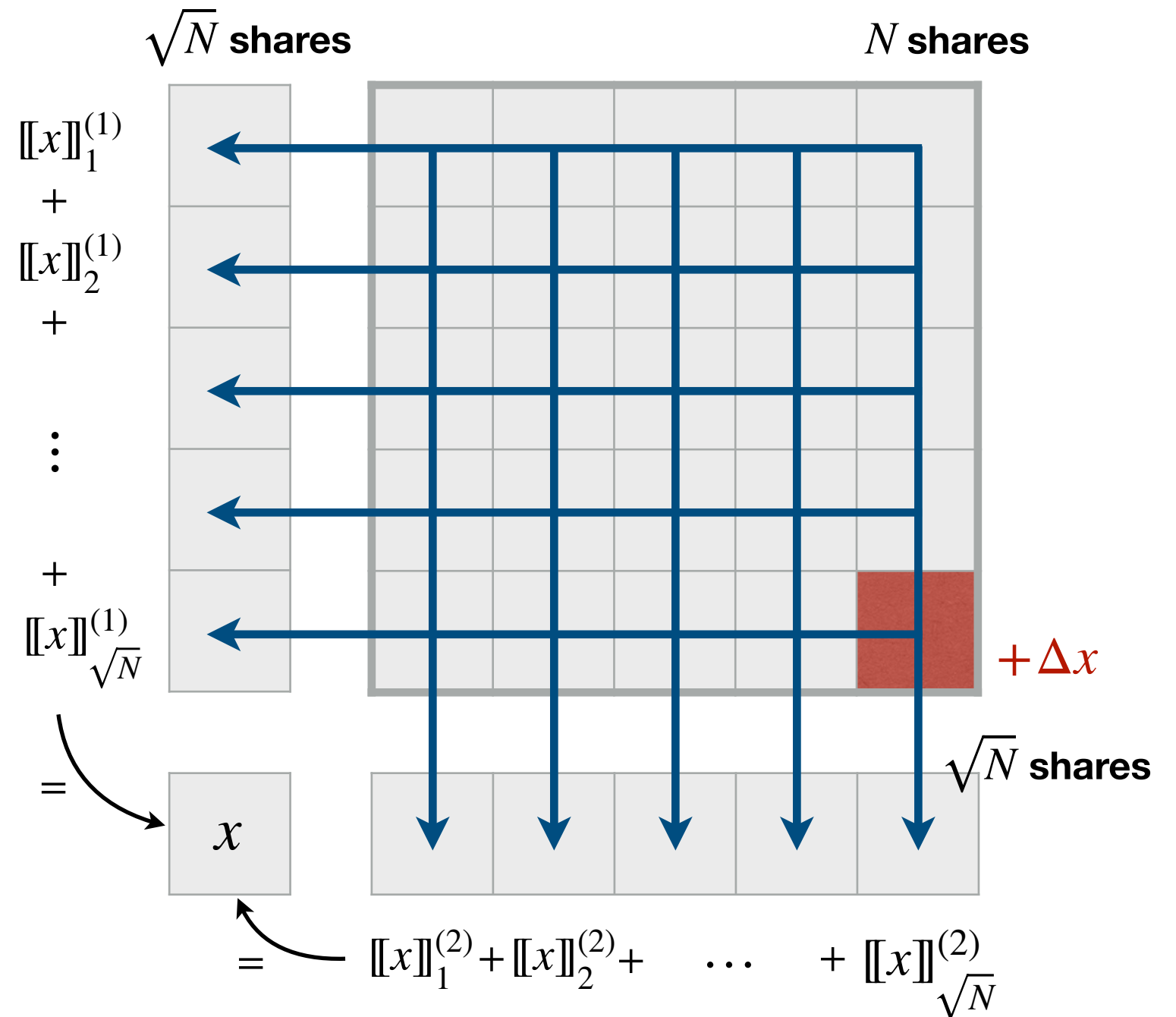
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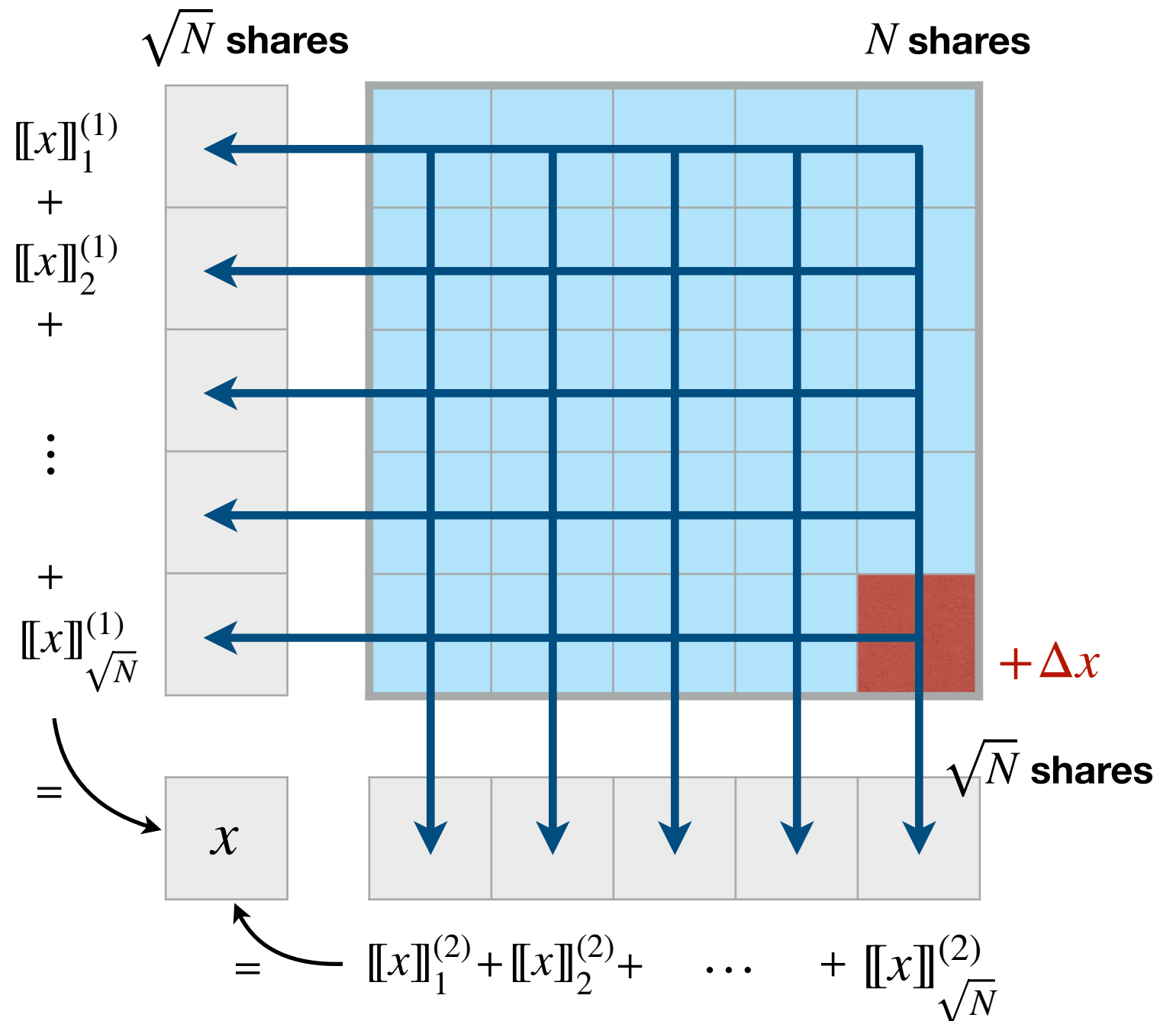


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Traditional approach:

- Emulating the N -party protocol with inputs $[[x]]_1, \dots, [[x]]_N$
- Chance of cheating $1/N$



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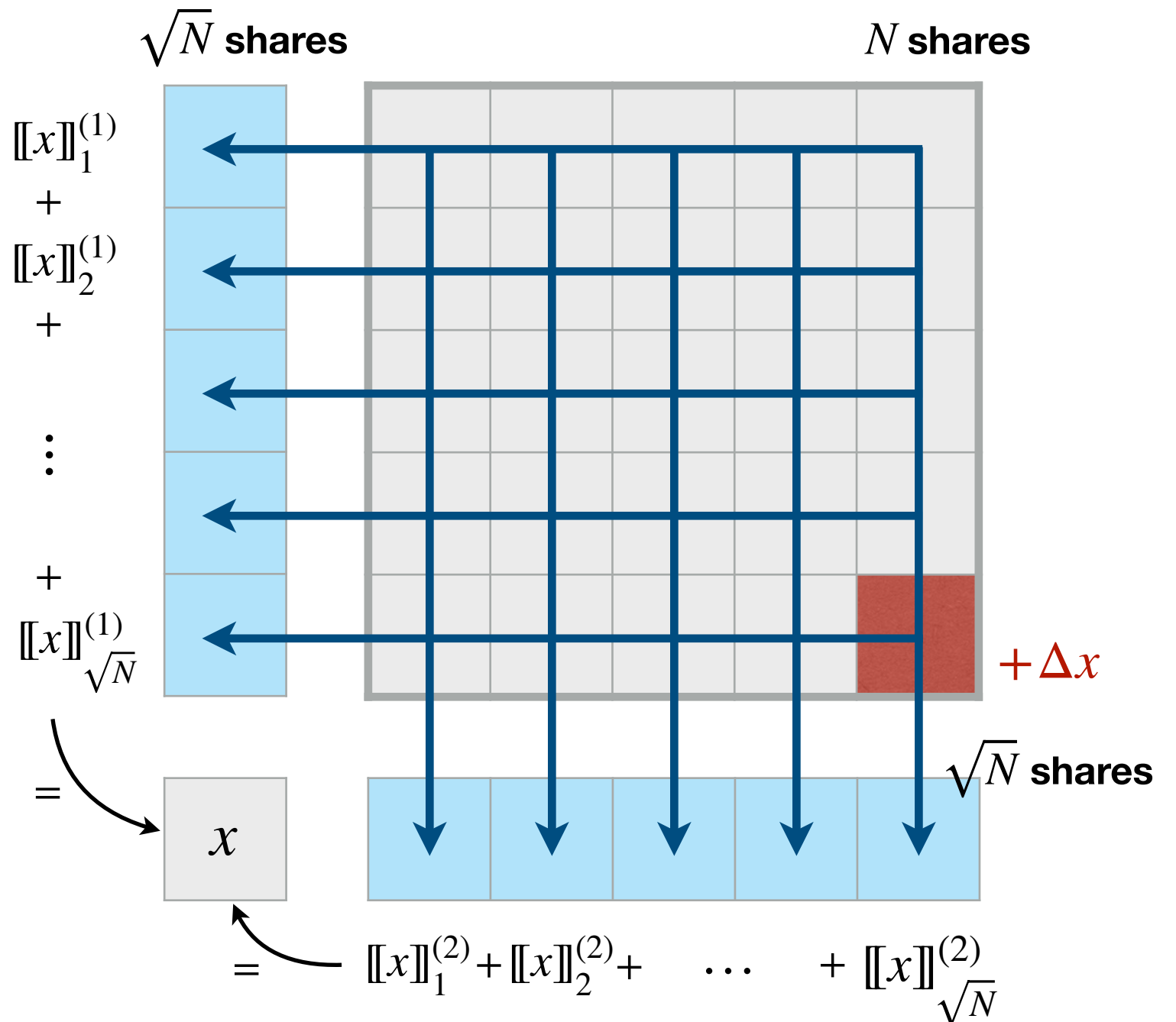
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- Emulating the \sqrt{N} -party protocol with inputs $[[x]]_1^{(1)}, \dots, [[x]]_{\sqrt{N}}^{(1)}$
- Emulating the \sqrt{N} -party protocol with inputs $[[x]]_1^{(2)}, \dots, [[x]]_{\sqrt{N}}^{(2)}$

- Chance of cheating

$$\left(\frac{1}{\sqrt{N}}\right)^2 \rightarrow \frac{1}{N}$$



The Hypercube Technique

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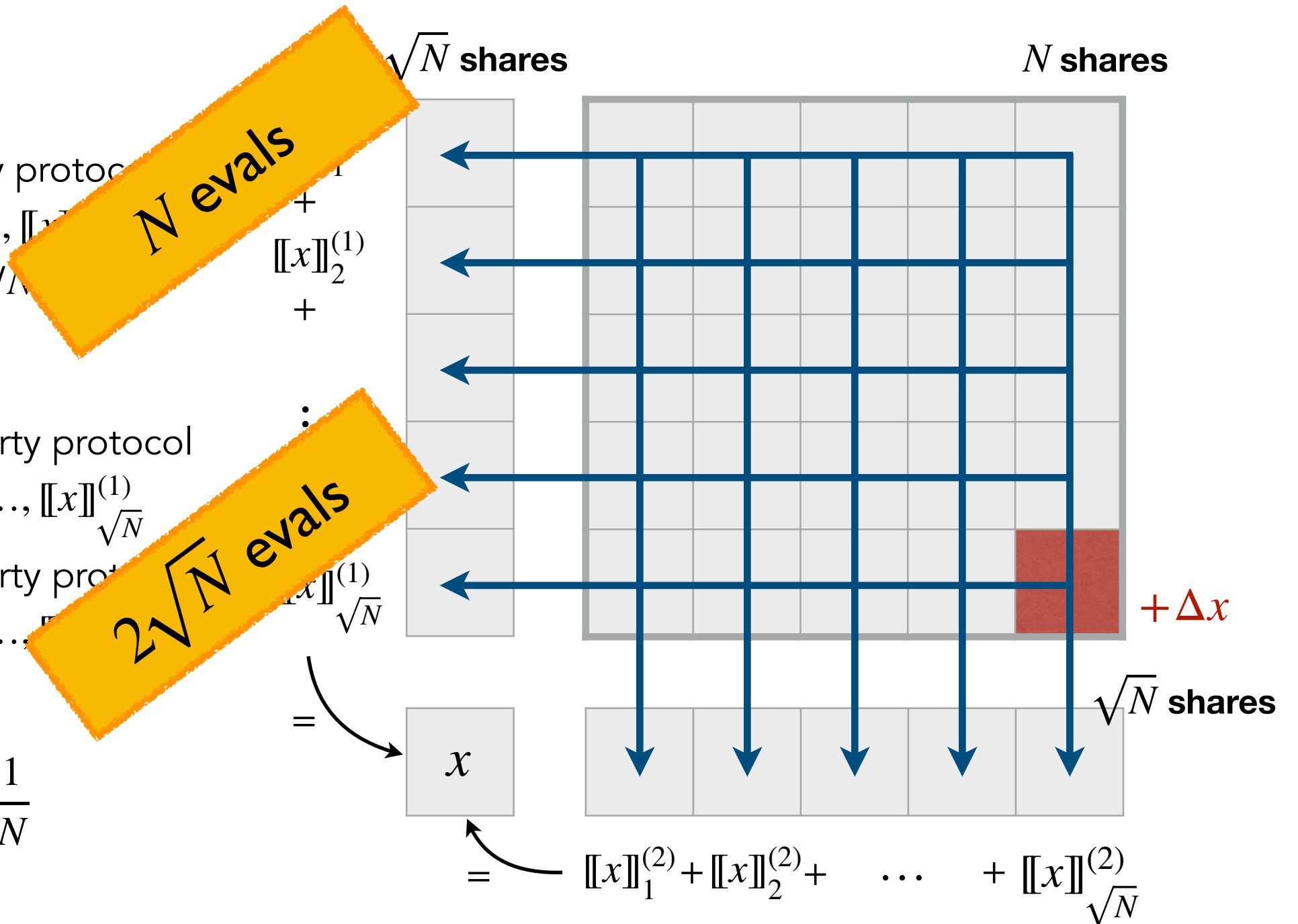
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The Hypercube Technique

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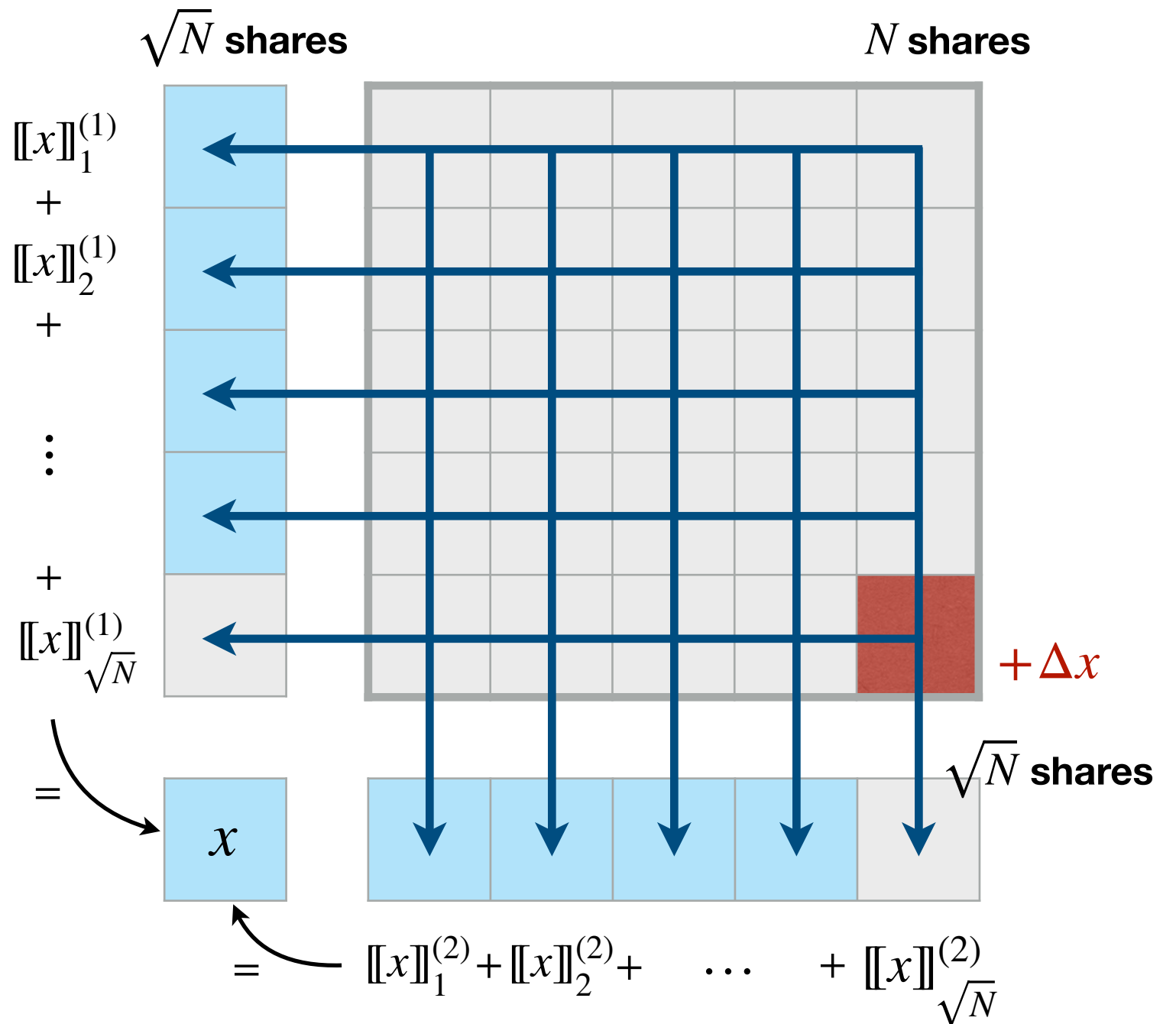
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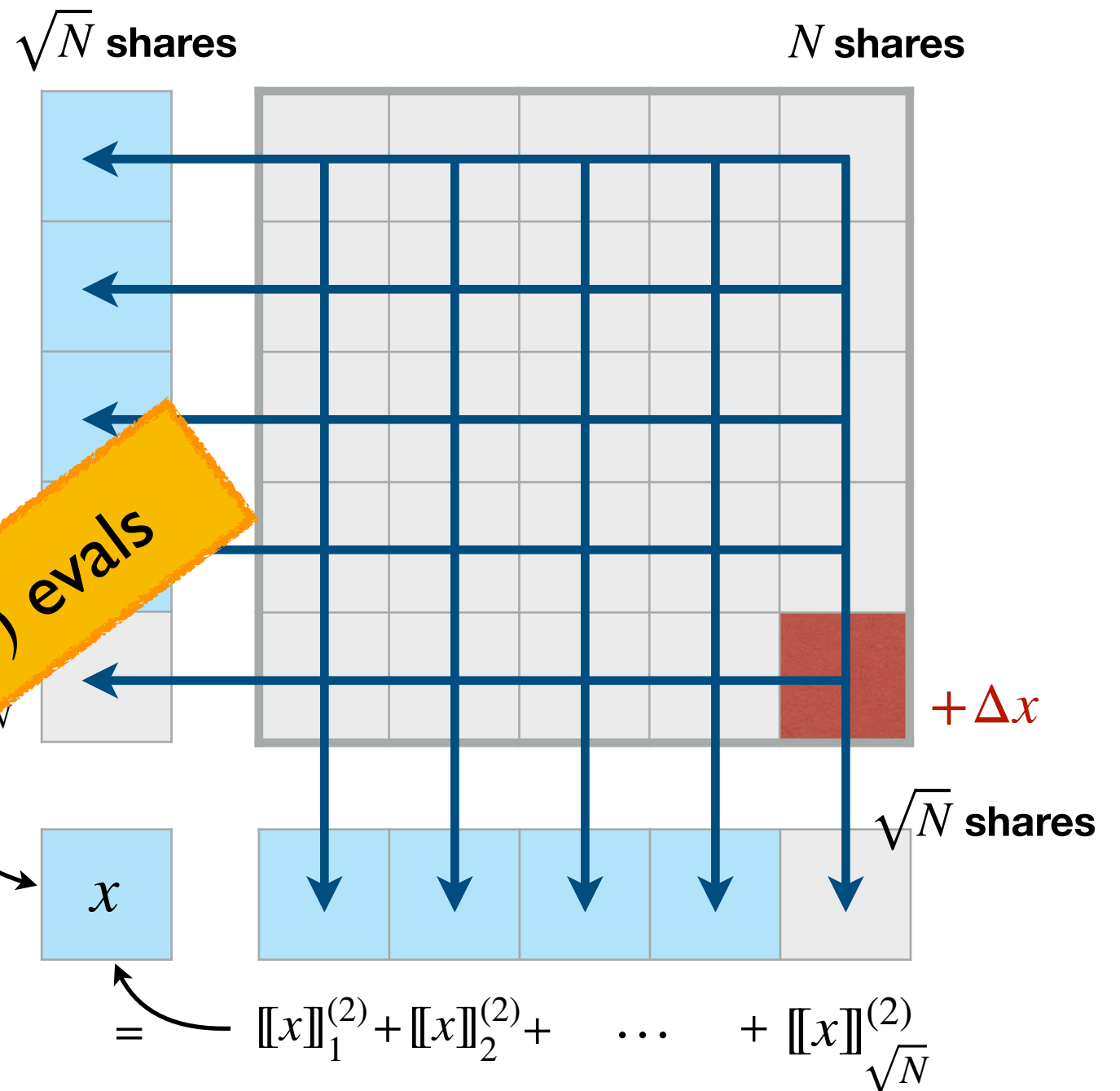
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$1 + 2(\sqrt{N} - 1)$ evals



The Hypercube Technique

[AGHHJY23] Aguilar-Melchor, Gama, Howe, Hülsing, Joseph, Yue: “The Return of the SDitH”
(Eurocrypt 2023)

Previous slide: square of side \sqrt{N}

The Hypercube Technique

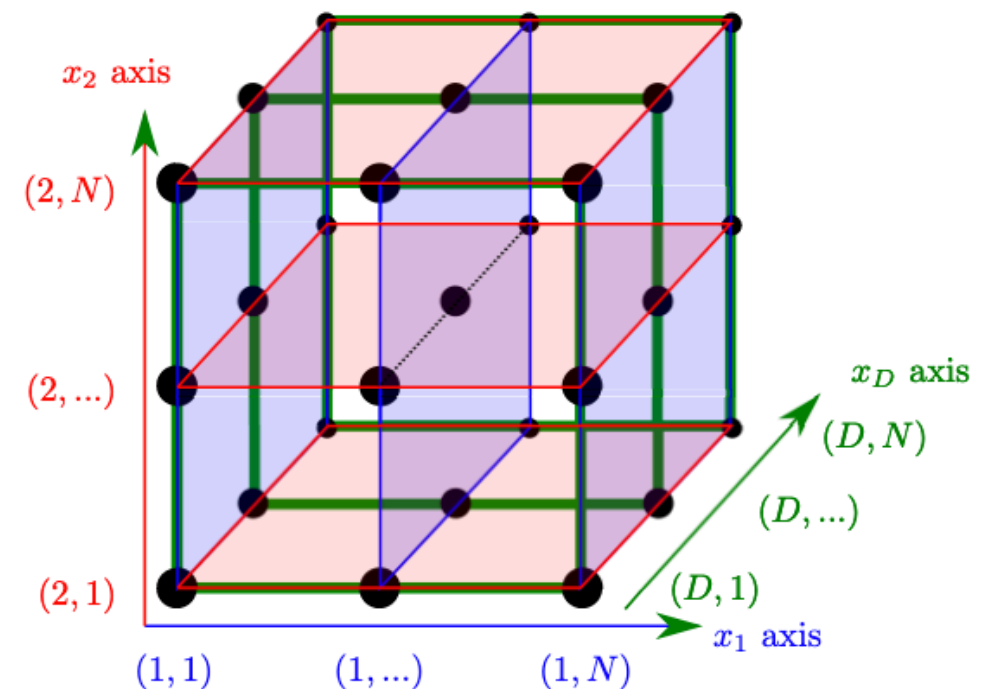
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The hypercube technique: hypercube of dimension $\log_2 N$ (each side has a size of 2)

Emulating $\log_2 N$ subprotocols with 2 parties.

Source: Figure from [AGHHJY23]



The $D \times N$ main party slices

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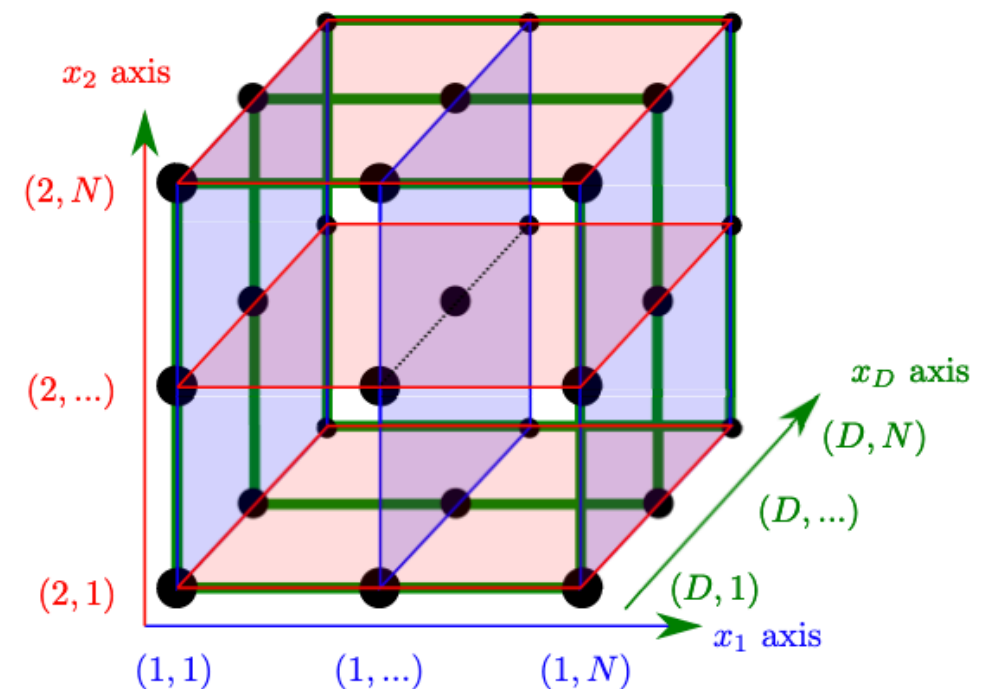
Soundness error:

$$\left(\frac{1}{2}\right)^{\log_2 N} = \frac{1}{N}$$

Emulation cost:

$$2 \cdot \log_2 N \text{ parties}$$

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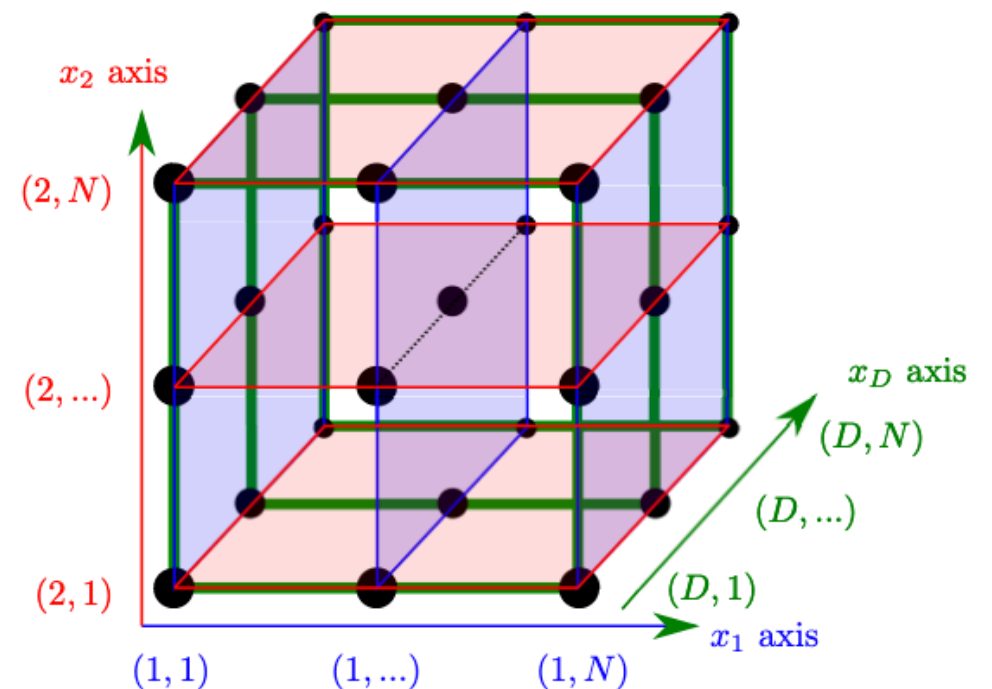
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$1 + \log_2 N$ parties

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Traditional: N party emulations per repetition

$$N = 256$$

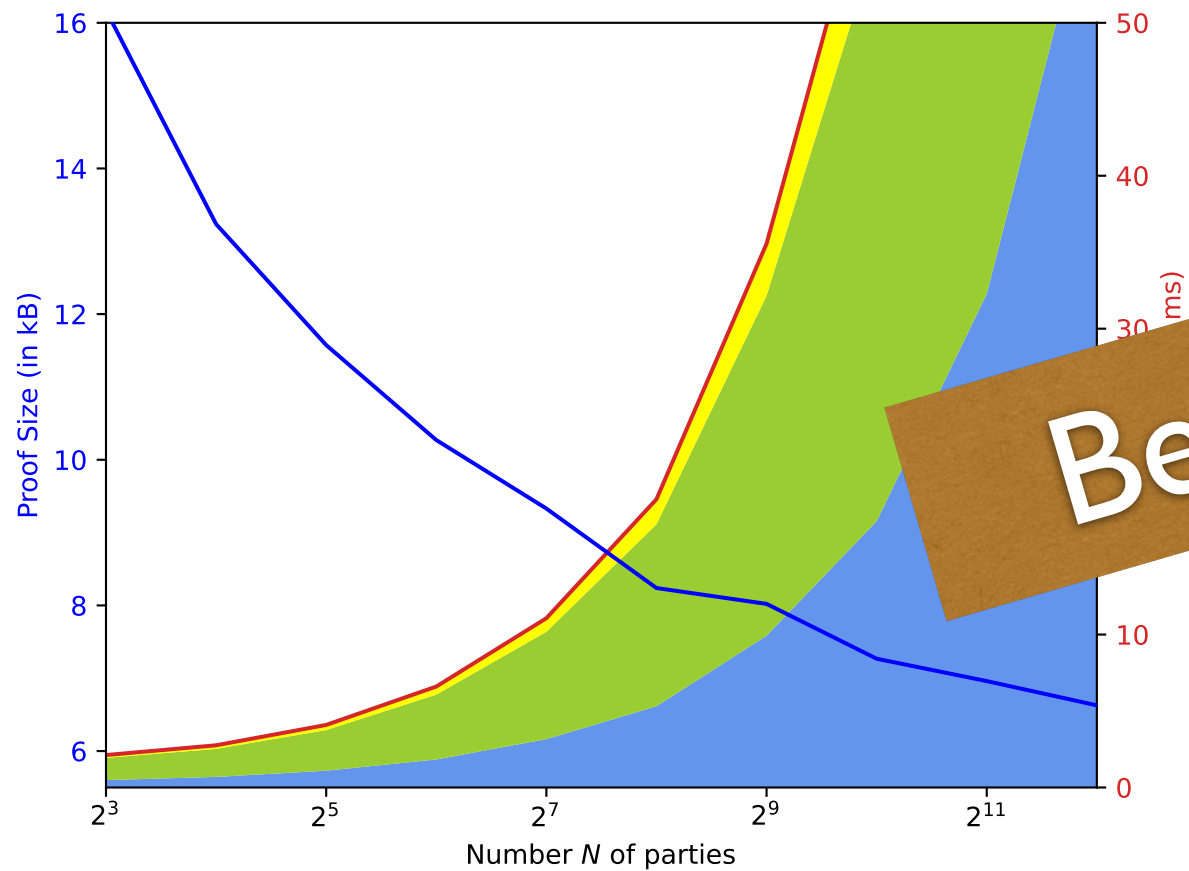


Hypercube: $1 + \log_2 N$ party emulations per repetition

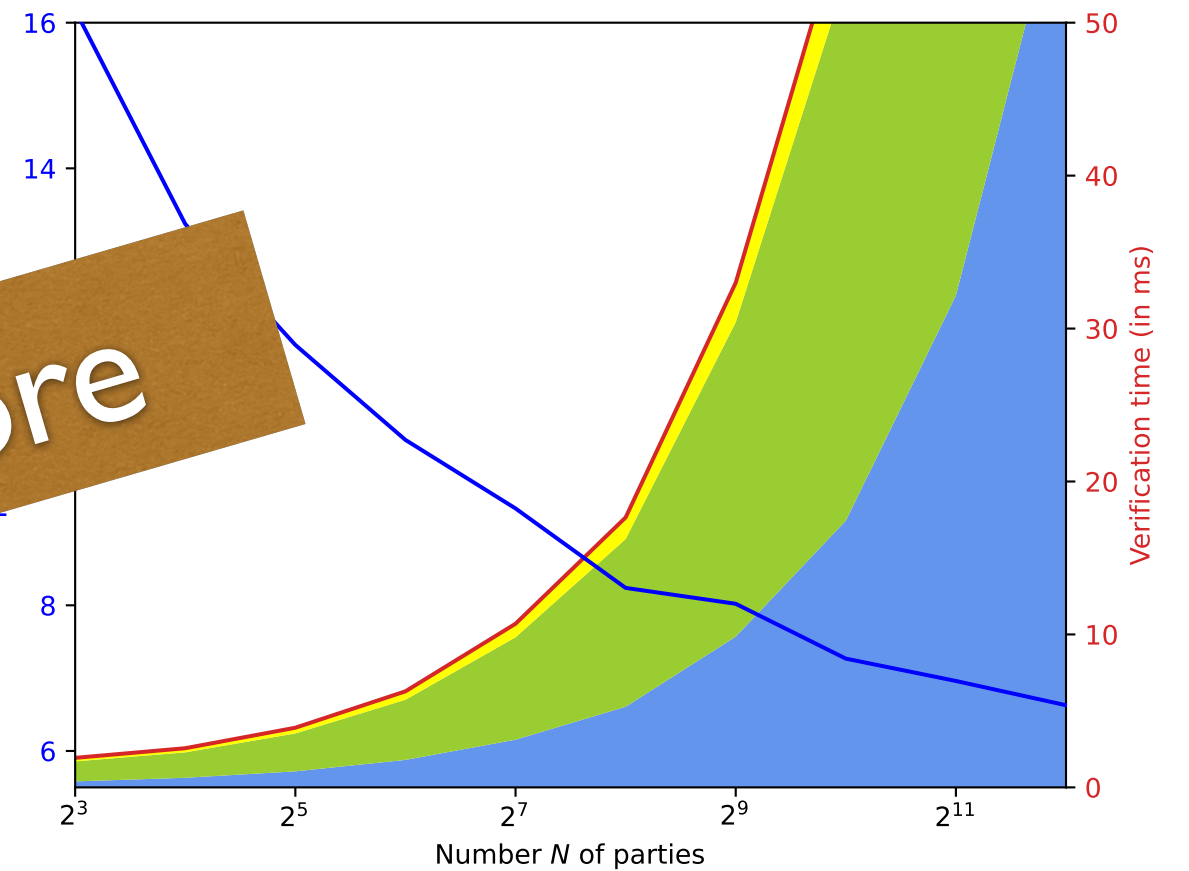
$$1 + \log_2 N = 9$$

The Hypercube Technique

Signing algorithm



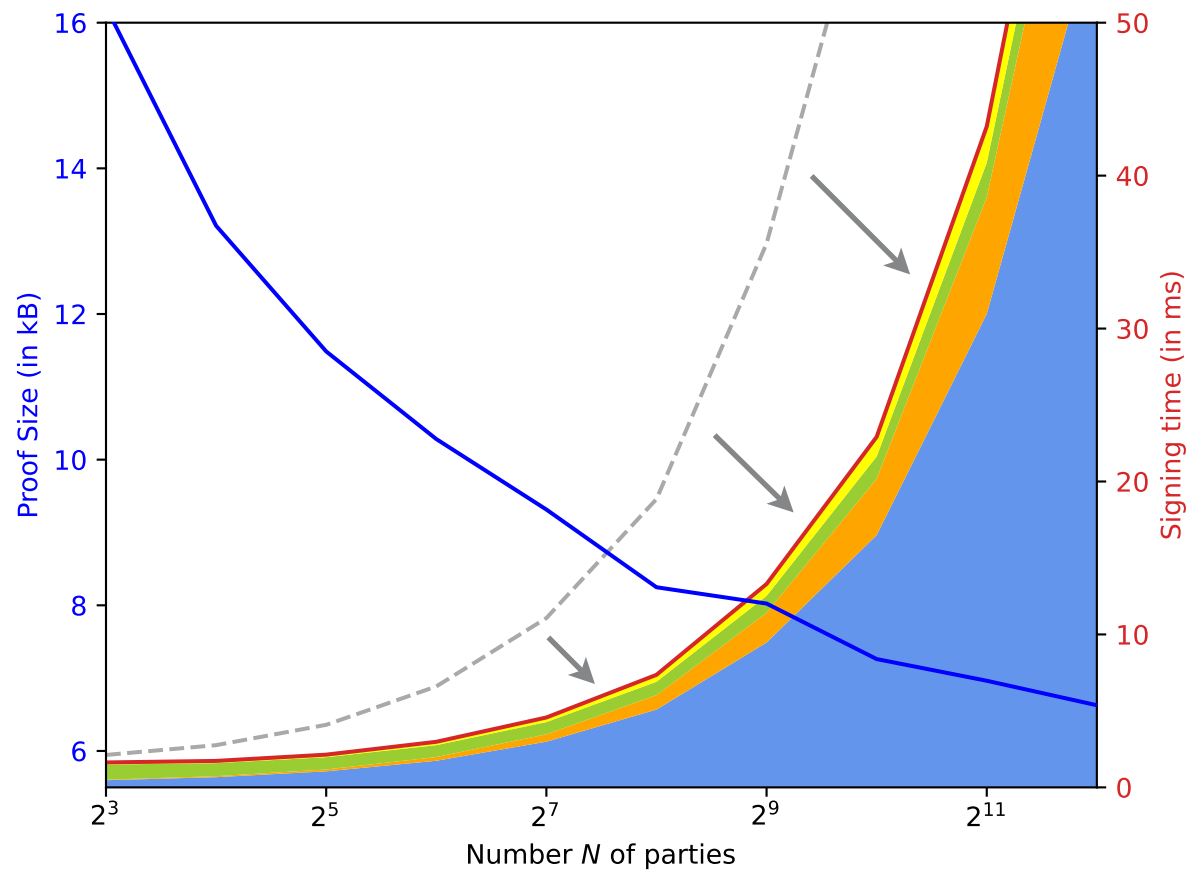
Verification algorithm



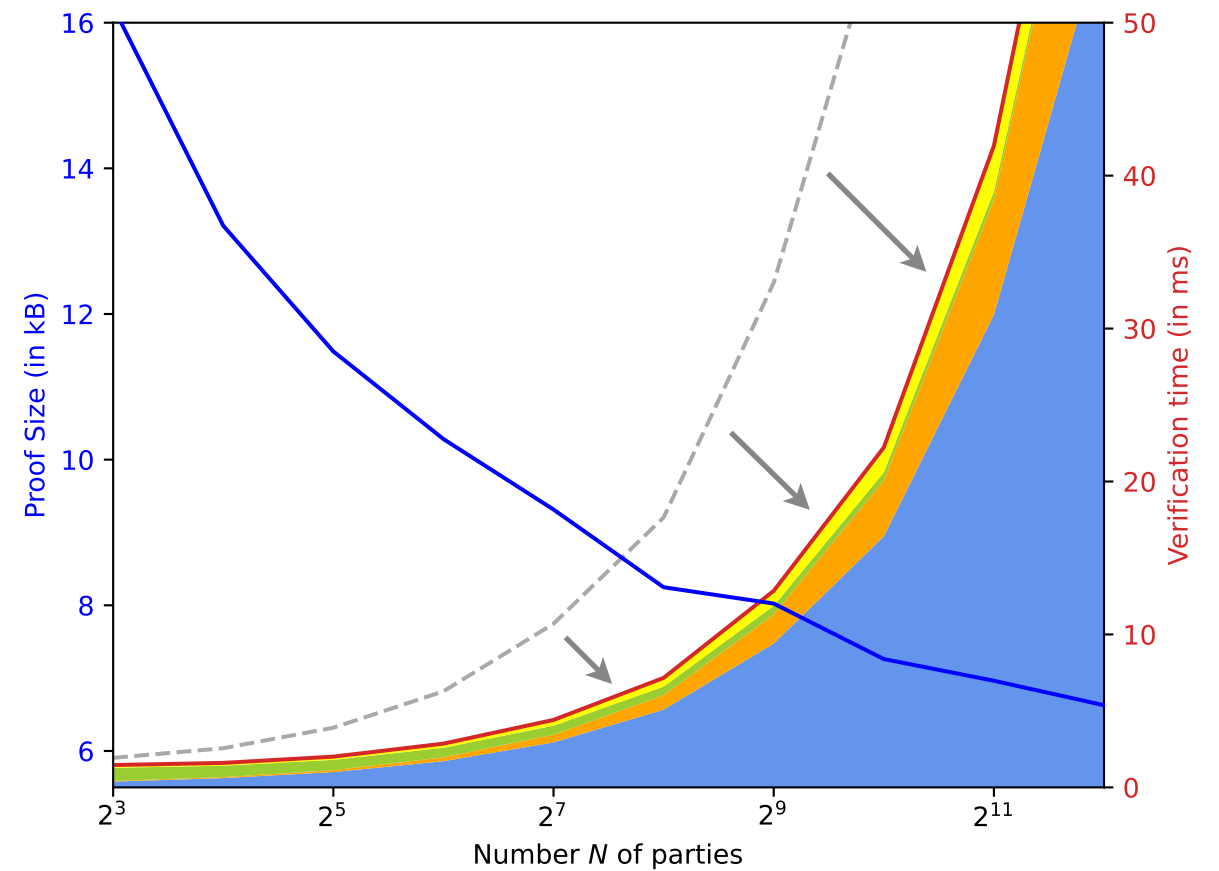
Before

The Hypercube Technique

Signing algorithm



Verification algorithm

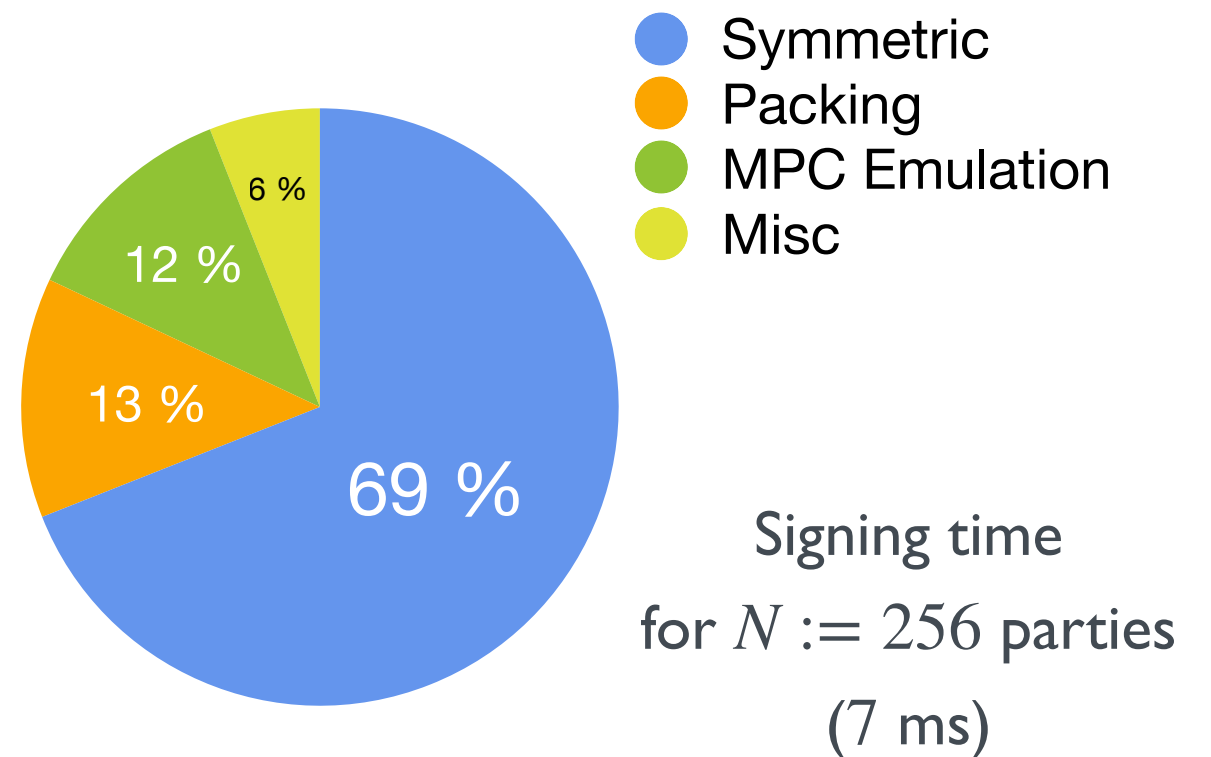
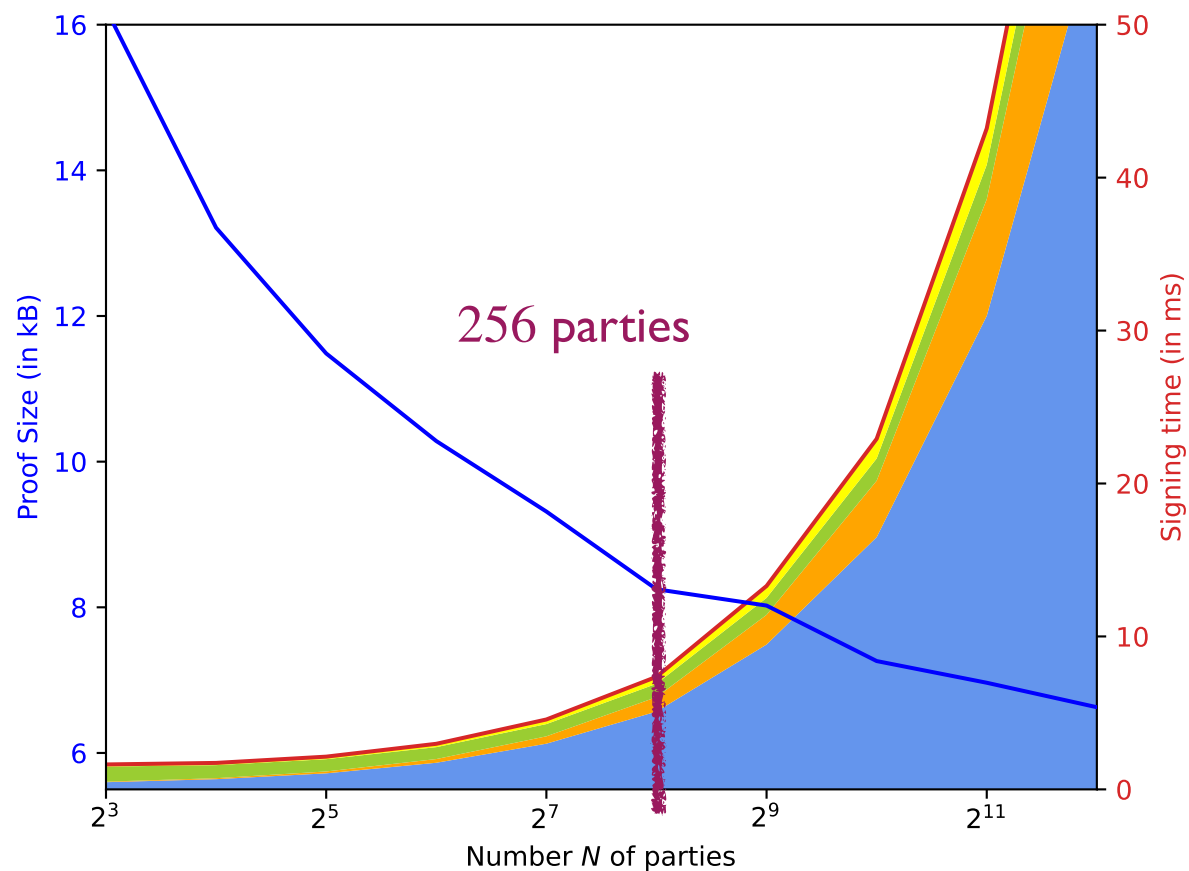


- Symmetric
- Packing
- MPC Emulation
- Misc

Running times @3.80Ghz

The Hypercube Technique

Signing algorithm



Running times @3.80Ghz

The Threshold Approach

[FR22] Feneuil, Rivain: "Threshold Linear Secret Sharing to the Rescue of MPC-in-the-Head"
(ePrint 2022/1407)

In the *threshold* approach, we used an **low-threshold** sharing scheme. For example, the Shamir's (ℓ, N) -secret sharing scheme.

To share a value x ,

- sample r_1, r_2, \dots, r_ℓ uniformly at random,
- build the polynomial $P(X) = x + \sum_{k=0}^{\ell} r_k \cdot X^k$,
- Set the share $[[x]]_i \leftarrow P(e_i)$, where e_i is publicly known.

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For example, the Shamir’s (ℓ, N) -secret sharing scheme.

The prover reveals only ℓ shares to the verifier (instead of $N - 1$).

In practice, $\ell \in \{1,2,3\}$.

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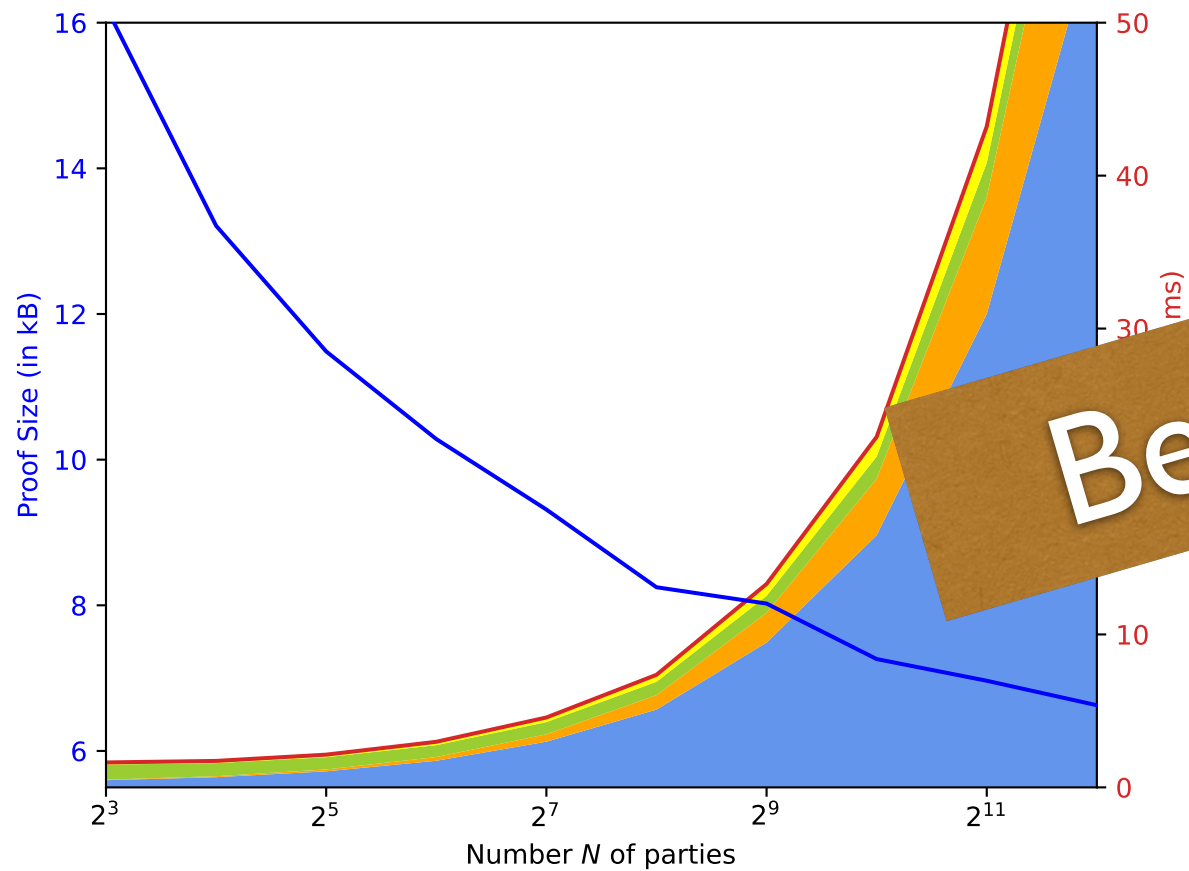
In practice, $\ell \in \{1,2,3\}$.

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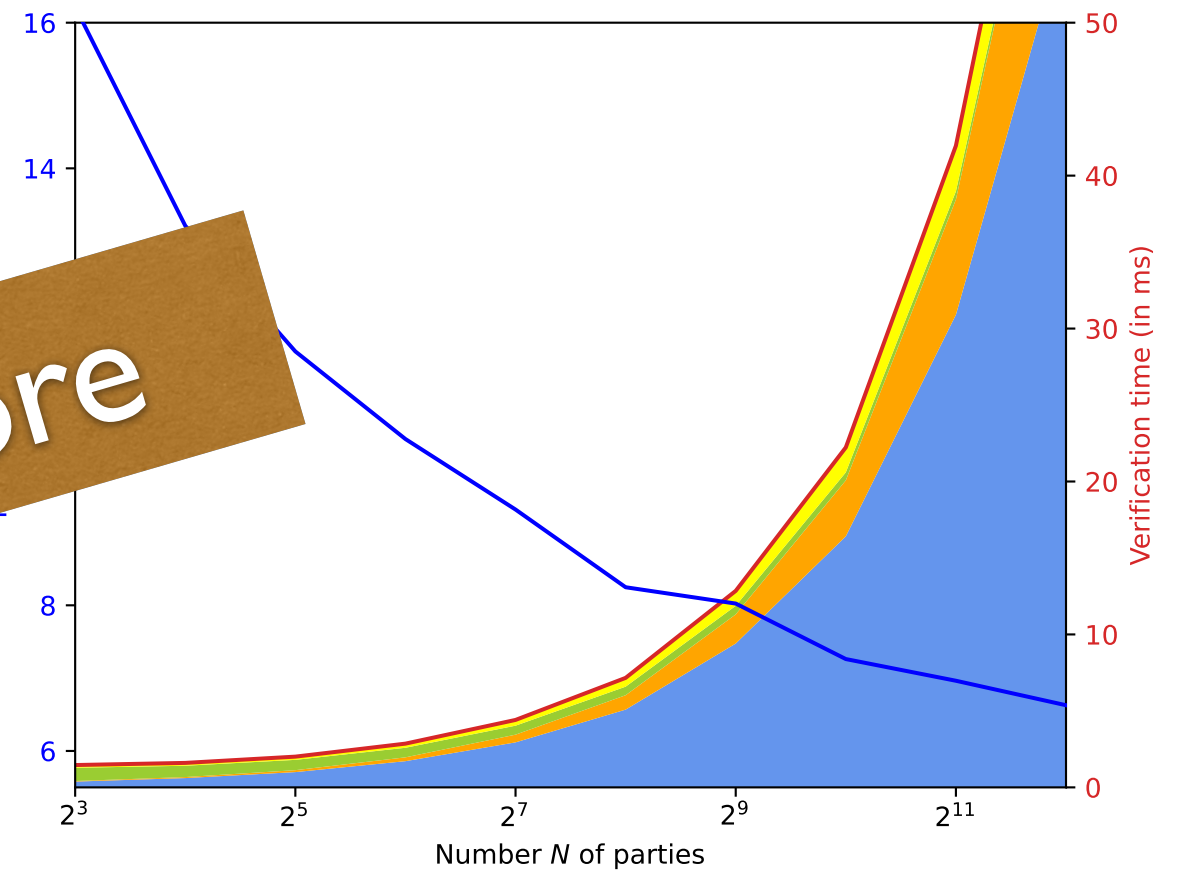
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- The obtained signature size is **larger**;
- We have the constraint: $N \leq |\mathbb{F}|$.

The Threshold Approach

Signing algorithm

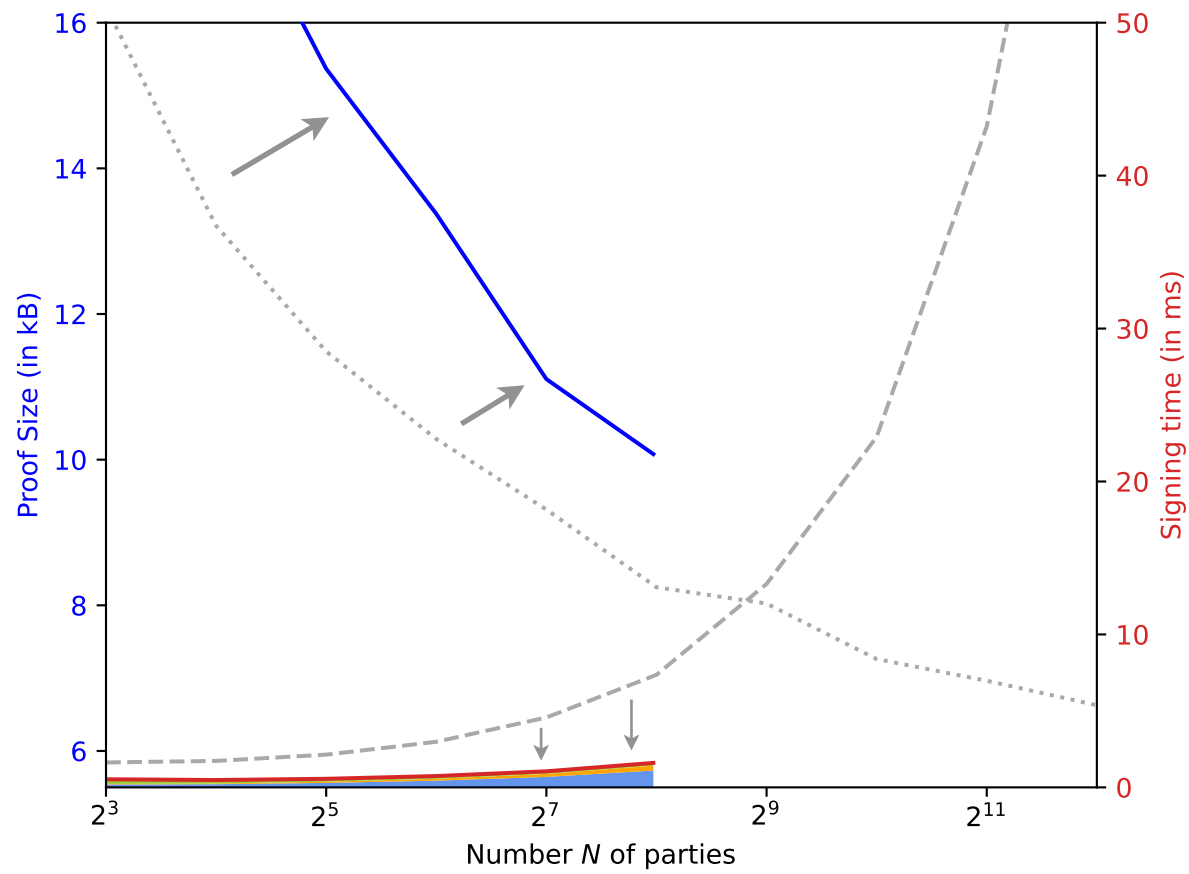


Verification algorithm

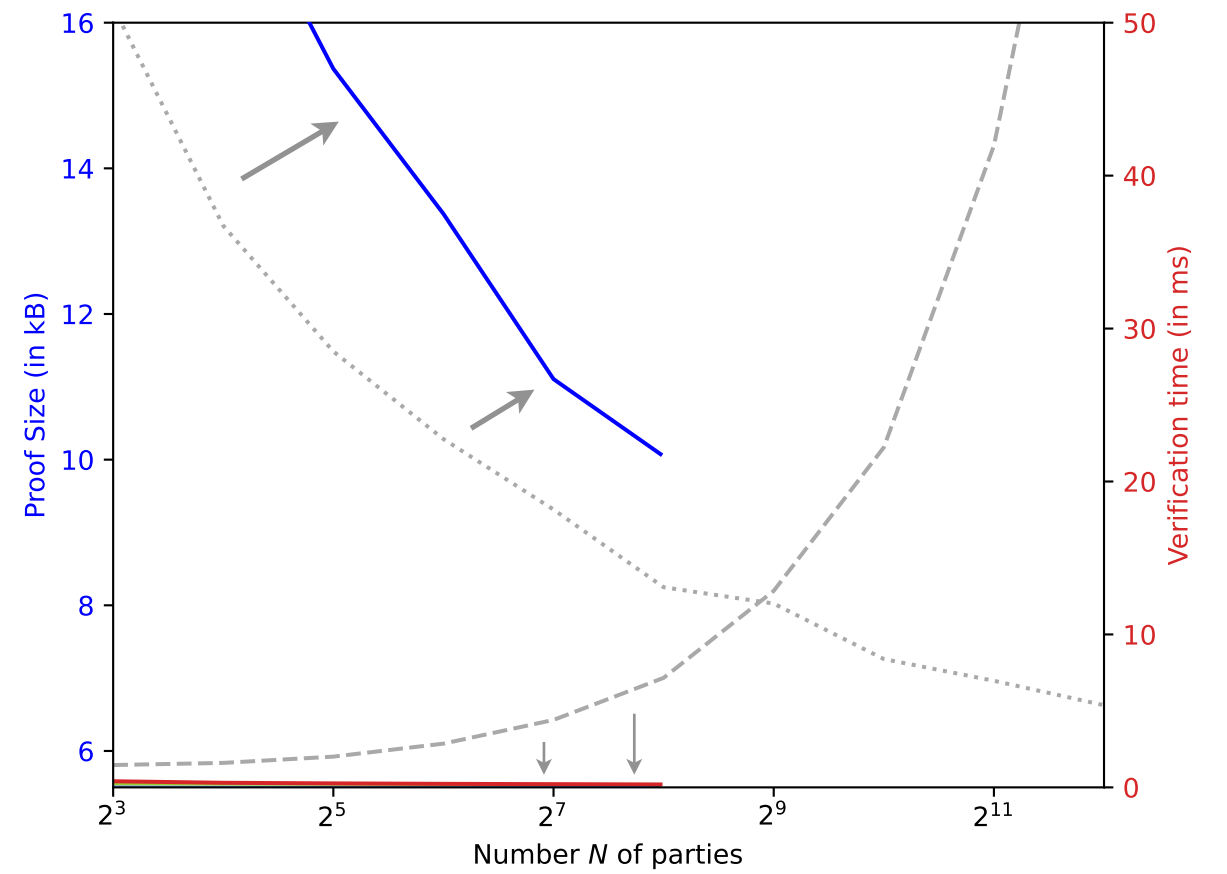


The Threshold Approach

Signing algorithm



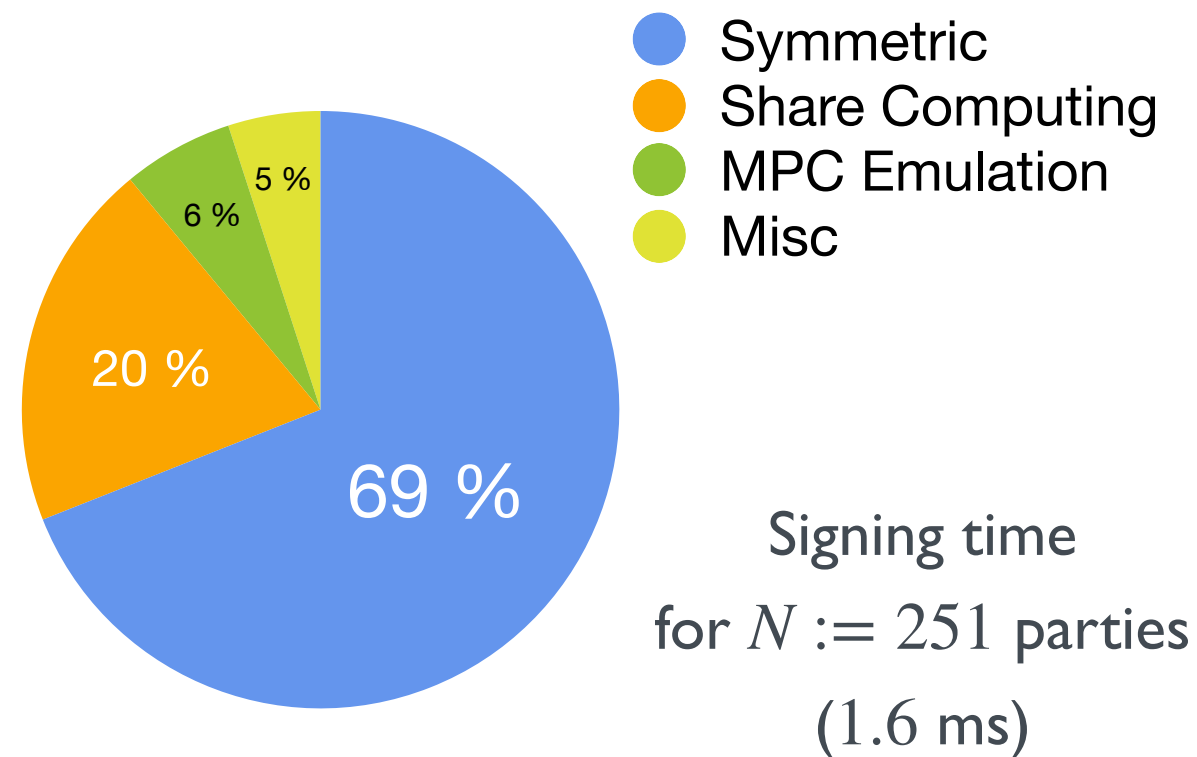
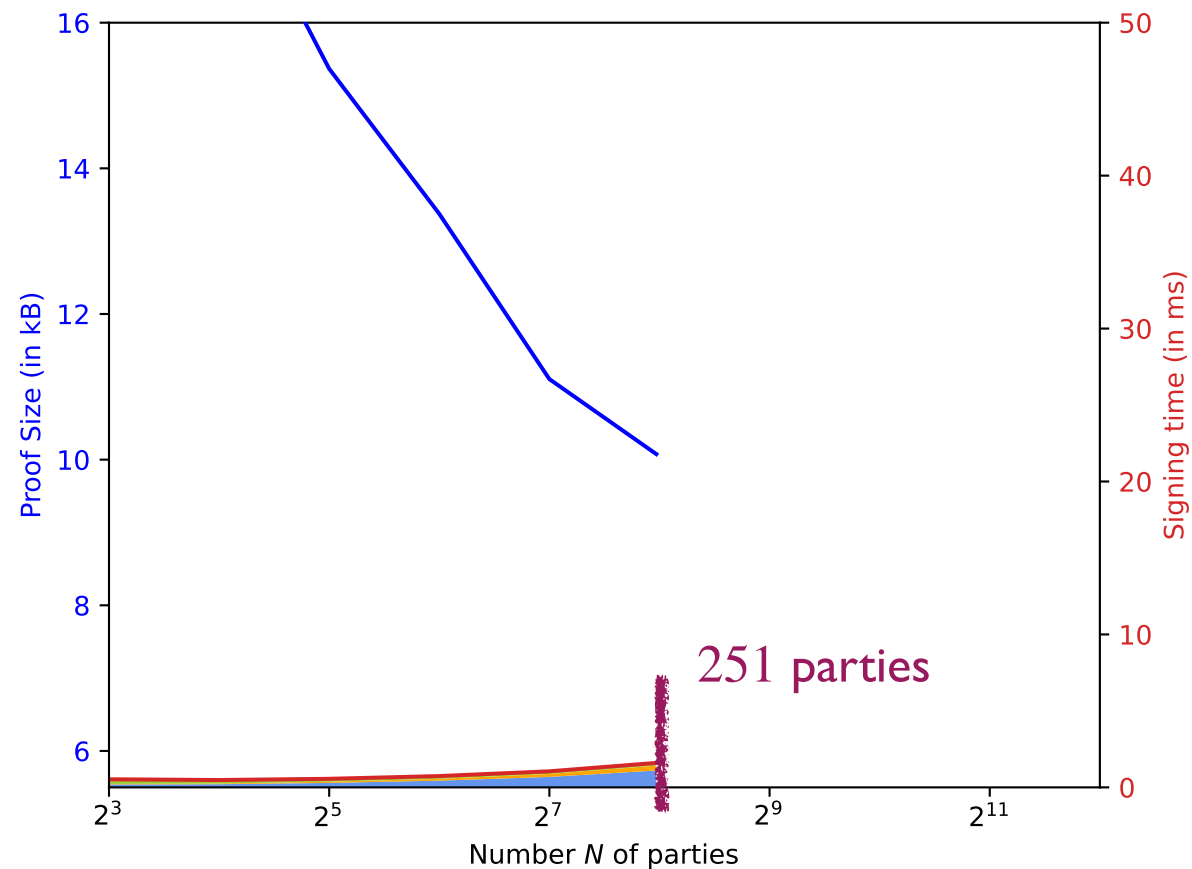
Verification algorithm



Running times @3.80Ghz

The Threshold Approach

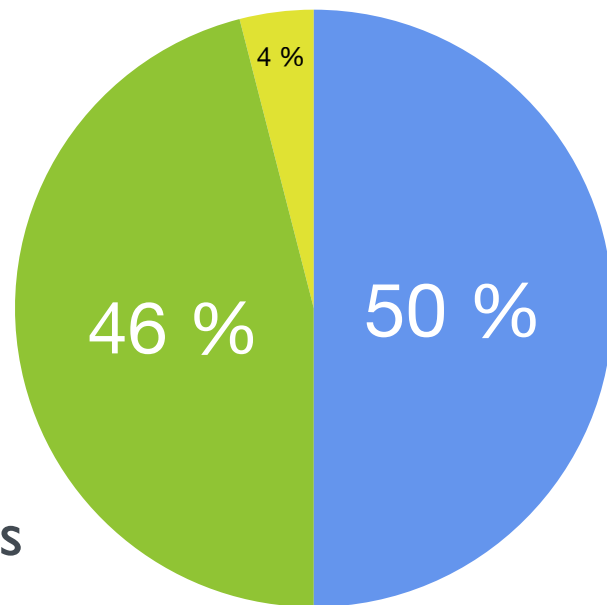
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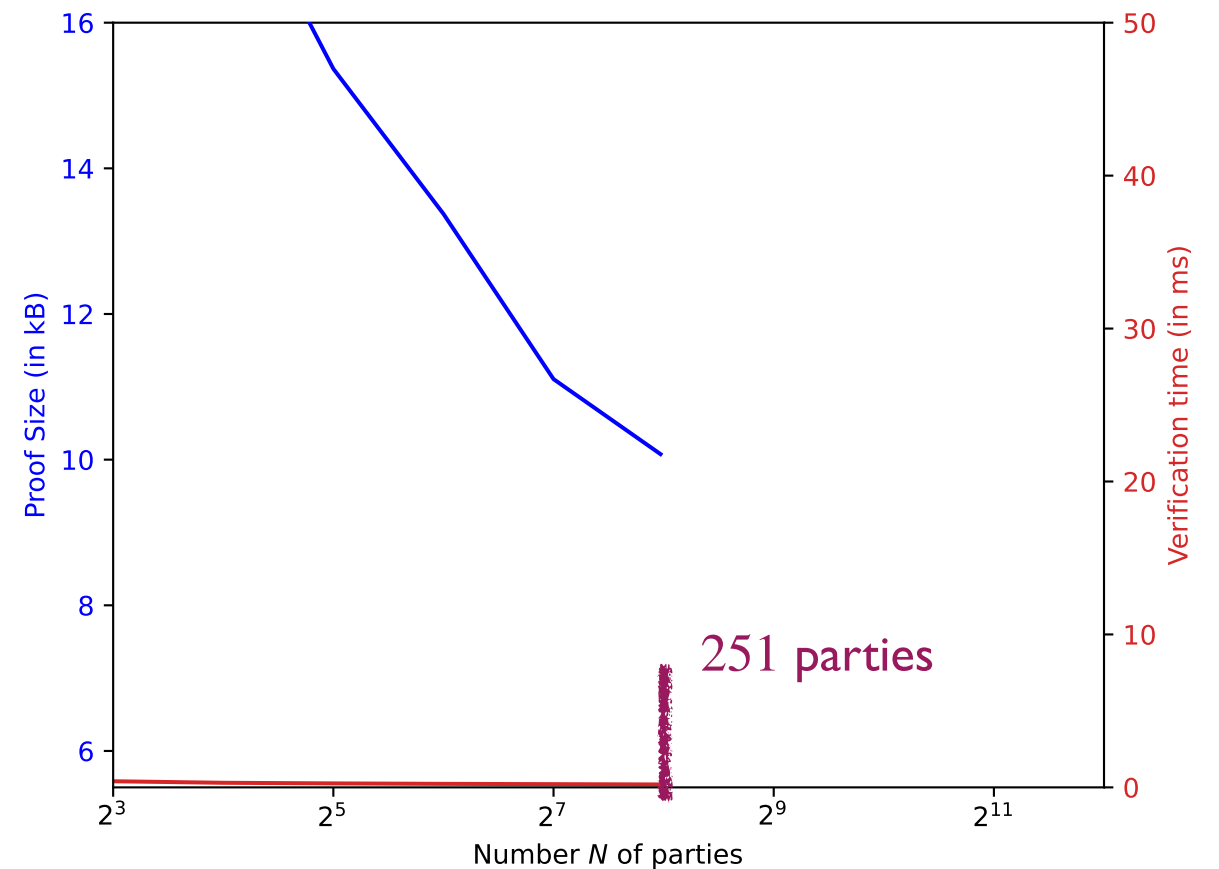
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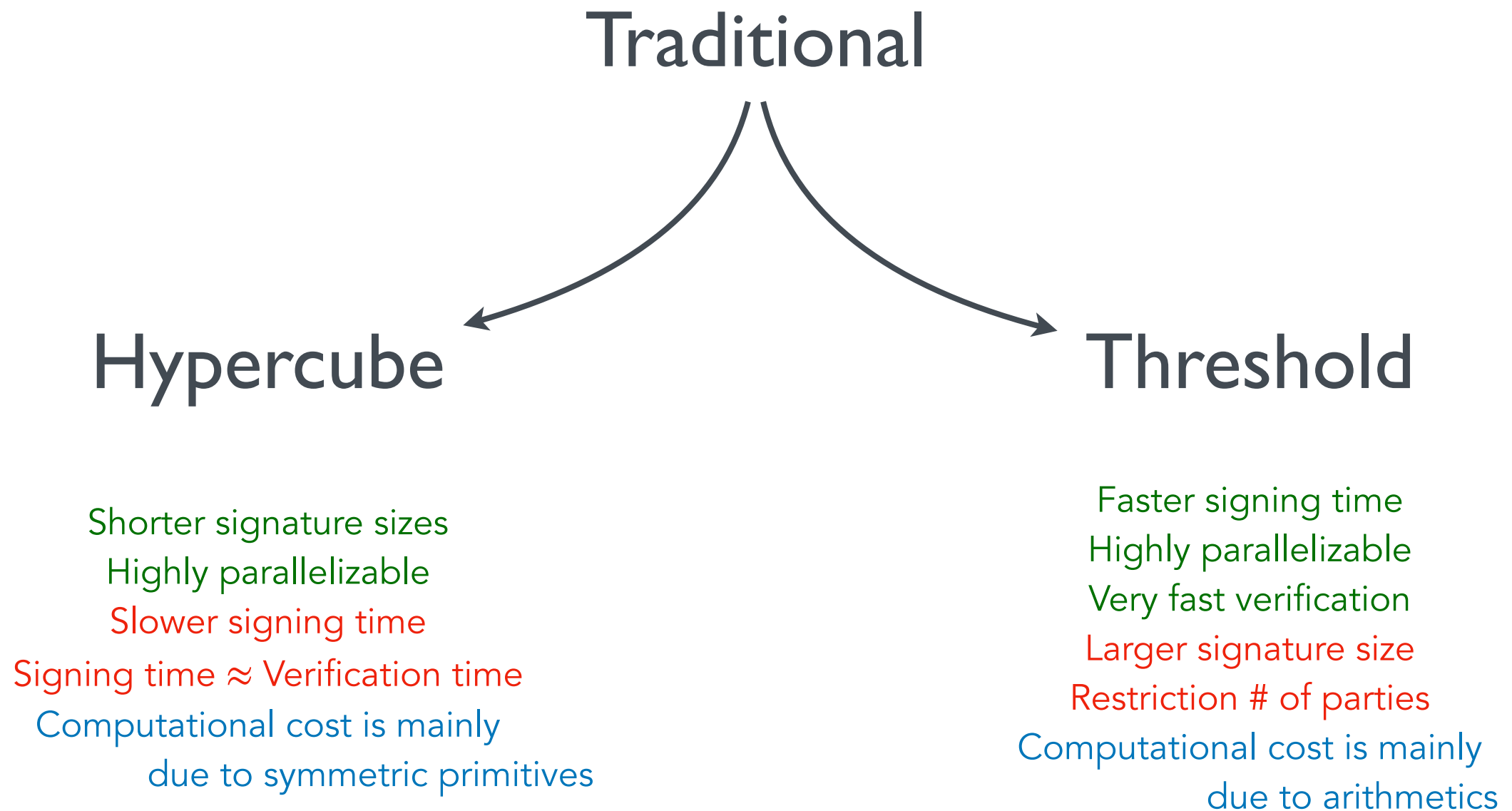
Verification time
for $N := 251$ parties
(0.2 ms)

Running times @3.80Ghz

Verification algorithm



The existing MPCitH transforms



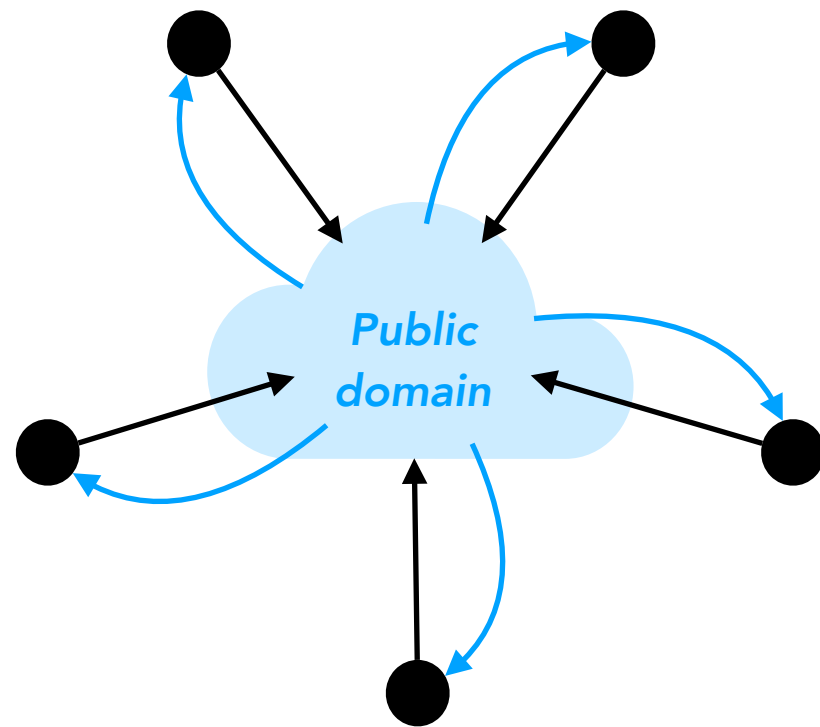
MPCitH-based NIST candidates

	Short Instance	Fast Instance
AlMer	Traditional (256-1615)	Traditional (16-57)
Biscuit	Traditional (256)	Traditional (16)
MIRA	Hypercube (256)	Hypercube (32)
MiRith	Traditional (256)	Traditional (16)
	Hypercube (256)	Hypercube (16)
MQOM	Hypercube (256)	Hypercube (32)
RYDE	Hypercube (256)	Hypercube (32)
SDitH	Hypercube (256)	Threshold (251-256)

Related works

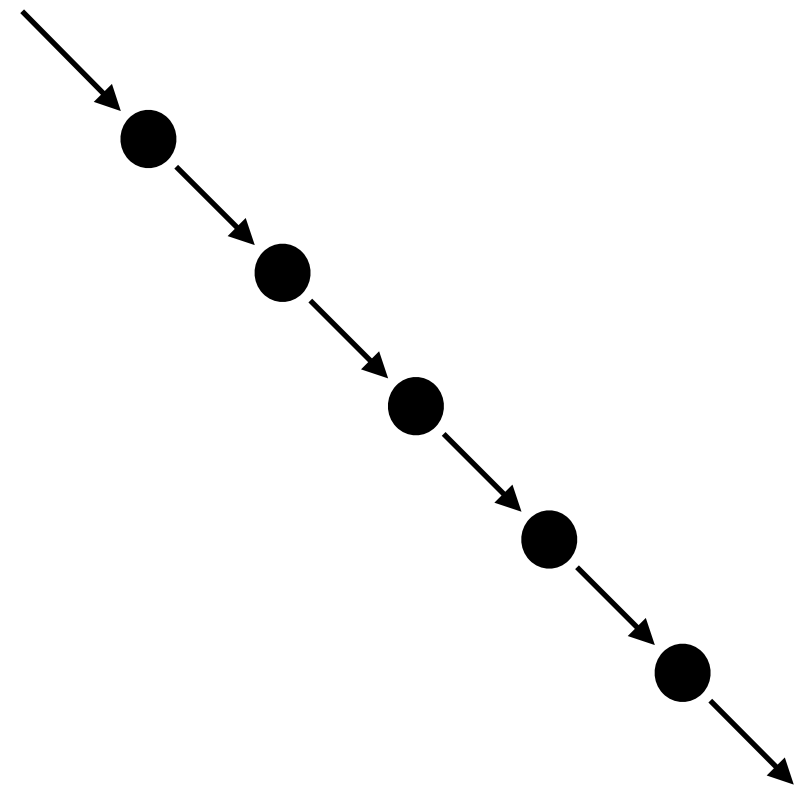
PERK: Shared Permutation on Permuted Kernel Problem

Standard MPC-in-the-Head



AlMer, Biscuit, MIRA, MiRitH
MQOM, RYDE, SDitH

Path-based MPC-in-the-Head



PERK

FAEST: VOLE-in-the-Head

VOLE: *vector oblivious linear evaluation*

“FAEST is the first AES-based signature scheme to be smaller than SPHINCS+”

Will be presented at Crypto'23 the 23rd August

Conclusion

Advantages and limitations

■ Limitations

- Relatively **slow** (*few milliseconds*)
 - Greedy use of symmetric cryptography
- Relatively **large** signatures (*4-10 KB for LI*)
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■ Advantages

- **Conservative** hardness assumption:
 - No structure (often), no trapdoor
- **Small** (public) keys
- **Good** public key + signature size
- Adaptive and **tunable** parameters

Conclusion

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 - Can be applied on any one-way function
 - A practical tool to build *conservative* signature schemes

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Could lead to follow-up works

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Thank you for your attention.